

Rules for Exponents and Logarithms

Properties of Exponents

1. Exponentiation by Zero:

$$x^0 = 1 \qquad -x^0 = -1$$

2. Negative Exponents:

$$x^{-a} = \frac{1}{x^a} \qquad x^a = \frac{1}{x^{-a}} \qquad \frac{x^{-a}}{y^{-b}} = \frac{y^b}{x^a} \qquad \left(\frac{x}{y}\right)^{-a} = \left(\frac{y}{x}\right)^a$$

3. Product Rule:

$$x^a \cdot x^b = x^{a+b}$$

4. Quotient Rule:

$$\frac{x^a}{x^b} = x^{a-b} = \frac{1}{x^{b-a}}$$

5. Power Rules:

$$(x^a)^b = x^{a \cdot b} \qquad (xy)^a = x^a \cdot y^a \qquad \left(\frac{x}{y}\right)^a = \frac{x^a}{y^a}$$

$$\sqrt[a]{x^b} = x^{\frac{b}{a}} = \sqrt[a]{x^b}$$

NB: If n is odd, then $-x^n \neq (-x)^n$

Properties of Logarithms

1. Definition:

$n = \log_b(m)$ is, by definition, equivalent to $b^n = m$

Note: $\ln(x) = \log_e(x)$ and $\log(x) = \log_{10}(x)$

2. Zero Property:

$$\log_b(1) = 0$$

3. Identity Property:

$$\log_b(b) = 1$$

4. Inverse Rules:

$$\log_b(b^m) = m \qquad b^{\log_b(m)} = m$$

5. Product Rule:

$$\log_b(m \cdot n) = \log_b(m) + \log_b(n)$$

6. Quotient Rule:

$$\log_b\left(\frac{m}{n}\right) = \log_b(m) - \log_b(n)$$

7. Exponent Rule:

$$\log_b(m^a) = a \cdot \log_b(m)$$

8. Change of Base:

$$\log_b(m) = \frac{\log_a(m)}{\log_a(b)}$$