**Using Substitution**

**Homogeneous and Bernoulli Equations**

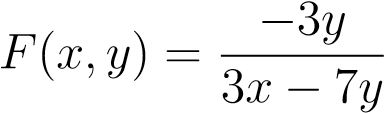
Sometimes differential equations may not appear to be in a solvable form. However, if we make an appropriate substitution, often the equations can be forced into forms which we can solve, much like the use of *u* substitution for integration. We must be careful to make the appropriate substitution. Two particular forms of equations lend themselves naturally to substitution.

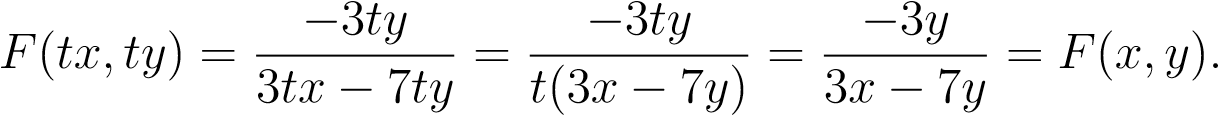
**Homogeneous Equations** A function *F*(*x,y*) is said to be *homogeneous* if for some *t* 6= 0

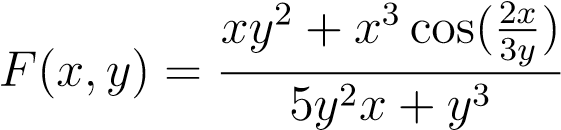
*F*(*tx,ty*) = *F*(*x,y*)*.*

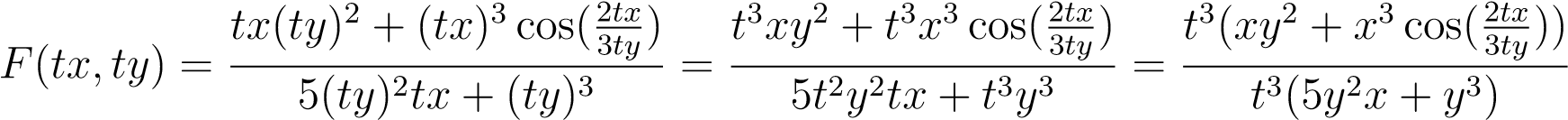
That is to say that a function is homogeneous if replacing the variables by a scalar multiple does not change the equation. Please note that the term homogeneous is used for two different concepts in differential equations.

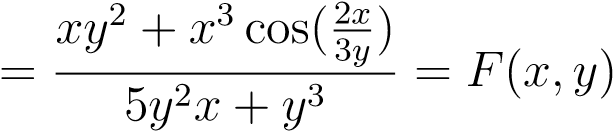
Examples

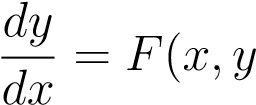
1.  is homogeneous since



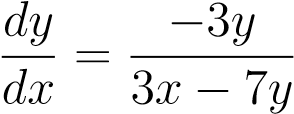
1.  is homogeneous since

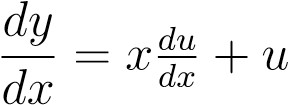


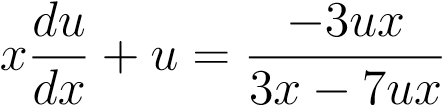


We say that a differential equation is homogeneous if it is of the form ) for a homogeneous function *F*(*x,y*). If this is the case, then we can make the substitution *y* = *ux*. After using this substitution, the equation can be solved as a *seperable* differential equation. After solving, we again use the substitution *y* = *ux* to express the answer as a function of *x* and *y*.

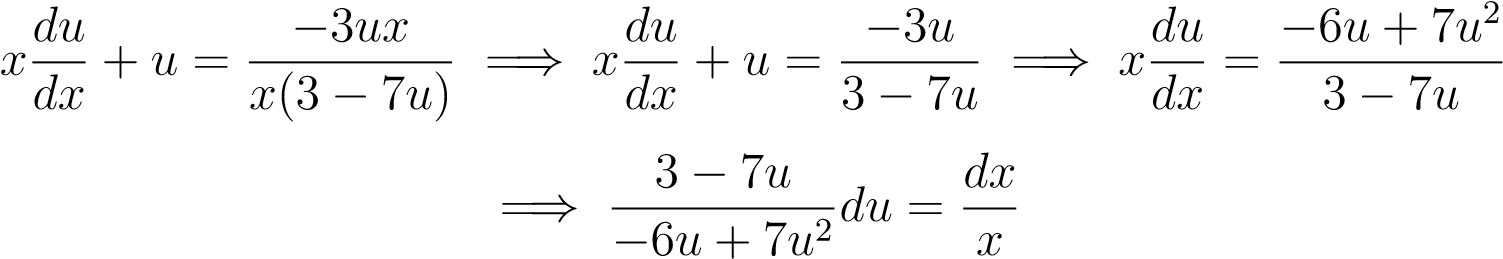
Example

1. 

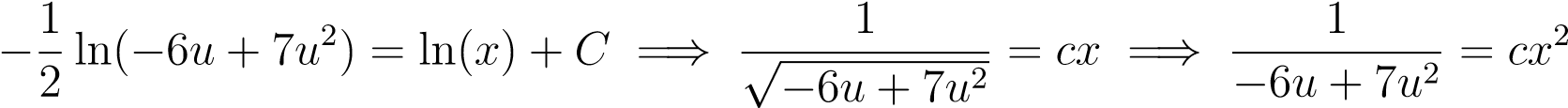
We have already seen that the function above is homogeneous from the previous examples. As a result, this is a homogeneous differential equation. We will substitute *y* = *ux*. By the product rule, . Making these substitutions we obtain



Now this equation must be separated.

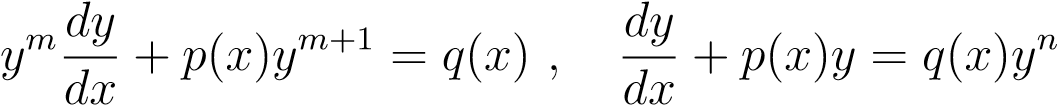


Integrating this we get,

*.*

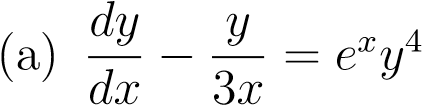
Finally we use thatto get our implicit solution .

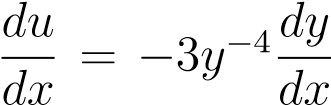
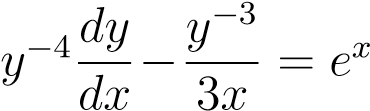
**Bernoulli Equations** We say that a differential equation is a *Bernoulli Equation* if it takes one of the forms

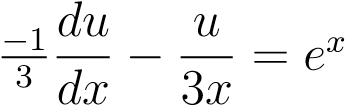
*.*

These differential equations almost match the form required to be linear. By making a substitution, both of these types of equations can be made to be linear. Those of the first type require the substitution *v* = *ym*+1. Those of the second type require the substitution *u* = *y*1−*n*. Once these substitutions are made, the equation will be *linear* and may be solved accordingly.

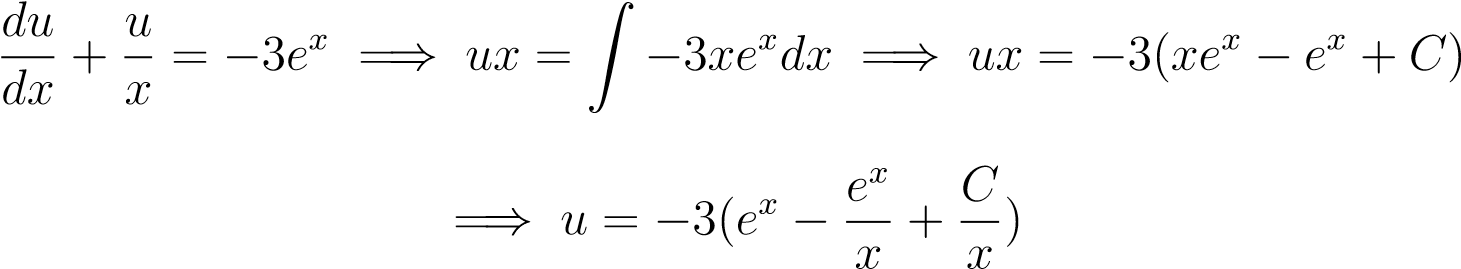
Example



You can see that this is a Bernoulli equation of the second form. We make the substitution *u* = *y*1−4 = *y*−3. This gives . The equation will be easier to manipulate if we multiply both sides by *y*−4. Our new equation will be.

Making the appropriate substitutions this becomes .

If we multiply by −3 we see that the equation is now linear in *u* and can be solved:



After undoing the *u* substitution, we have the solution

