Problem Solving Strategies

It can be overwhelming when we see a word problem, we might not be sure what the question is asking for, or what formula we should use, and it's common to just say, "I don't know where to start!!!"

So…

Where do we start?

Step 1: Read the Problem Carefully and Identify What the Problem is Asking.

- Identify what information is given in the word problem and what exactly it is asking us to find.
- What information is missing from the question that is needed to solve the problem?
- How does the problem relate to the concepts being covered in class, and what fundamentals apply to the given problem?
- Identify and write out any relevant formulas.

Tip: underline key phrases in the word problem, like "given", "find" or "solve for", as well as any given information like time, distance or speed.

Tip: Try to write out the relevant formulas from memory every time and check notes if stuck. This can aid with memorization.

Step 2: Name and Label All Information.

- In algebra we can represent unknown information using any variable we choose. For example, we can say distance $= \boldsymbol{d}$ or we can say distance $= \boldsymbol{x}$, as long as we are consistent with its use.
- When assigning variables, we typically choose to assign our variable to the element we know the least about. For example, if we have an example where Car A is traveling at an unknown speed, and Car B is traveling 15 mph faster than Car A. We know less about Car A than we do with Car B, so we can assign Car A's speed a variable and describe Car B's speed in terms of Car A's speed.
- For example, we can say Car A's speed $= \boldsymbol{A}$, and Car B's Speed $= \boldsymbol{A + 15}$.

Tip: Before we start solving the problem list all of the knowns, unknowns, and variables to refer back to them as the problem is solved.

It can also be useful to create a rough sketch and label the sketch with the corresponding data and variables.

Step 3: Describe The Steps in Words and Translate It to Algebra.

- What steps need to be taken to get from the given values provided in the question to the answer?
- For example, if we know Car A and Car B have a combined speed of 60 mph, we can describe that as: "the speed of Car A added to the speed of Car B is equal to 60 mph".
- Translating that to variables and symbols we get: $(\boldsymbol{A}) + (\boldsymbol{A + 15}) = \mathbf{60\ mph}$

Step 4: Solve the Equation.

- Using all of the algebra techniques learned in class, try to solve for all variables and check the solution against the original problem.

  Tip: When taking exams, make sure to prioritize time. It may be more efficient to come back and check the work after finishing answering the rest of the questions. However, if we know that we often make errors with order of operations or dropping negative signs, it may be beneficial to double check our work at every step.

Step 5: Describe What the Solution Means.

- Describe the solution in words, and state how it relates back to the original question.

Algebra Applications

Example: A carpenter must cut a 10ft board into two pieces to build a brace for a picnic table. If one piece needs to be four times longer than the other piece, how long should each piece be?

Step 1: Read the Problem Carefully and Identify What the Problem Is Asking For.

- What information has been given in the question?
- What information is missing from the question?
- What is the question asking us to solve for?

Reviewing the question again, underline the relevant information:

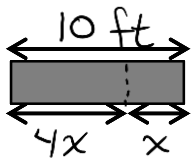
A carpenter must cut a **10ft board** into **two pieces** to build a brace for a picnic table. **If one piece needs** to be **four times longer than the other piece**, **how long** should **each piece** be?	
What information has been given?	The original piece of wood is 10ft long After cutting the larger piece needs to be four times longer than the shorter piece
What information is missing?	The length of the shorter piece The length of the longer piece
What is the question asking for?	The length of each piece.

Step 2: Name and Label the Information.

- For the unknown values we can choose any variables as long as we are consistent with their use.
- We know that the length of the long piece is four times longer than the length of the short piece. So, since we know less about the length shorter piece of wood than any other part, we can assign the variable x.
- And since we know that the length of the long piece is four times longer than the length of the short piece, we can translate that algebraically to $4 \cdot x$

Knowns	Unknowns
Original Length $= 10\ ft$	Short Piece Length $= x$
	Long Piece Length $= 4x$

Step 3: Describe the Steps in Words and Translate It to Algebra.



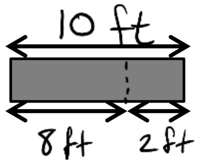
Describe	Translate
We know that the sum of the short piece and the long piece must be equal to the length of the original piece of wood	Short piece + Long piece = Original Length
We know the short piece has length x, the long piece has length $4x$, and that the original length of the wood is 10 ft	$x + 4x = 10ft$

Step 4: Solve the Equation.

$$\begin{aligned} 4x + x &= 10\ ft \\ 5x &= 10\ ft \\ \frac{5x}{5} &= \frac{10\ ft}{5} \\ x &= 2\ ft \end{aligned}$$

Knowns	Unknown	Answers
Original Length $= 10\ ft$	Short Piece Length $= x$	$x = 2\ ft$
	Long Piece Length $= 4x$	$4x = 4 \cdot 2 = 8\ ft$

Step 5: Describe What the Solution Means.



Example: A carpenter must cut a 10ft board into two pieces to build a brace for a picnic table. If one piece needs to be four times longer than the other piece, how long should each piece be?

Answer: In order to cut a 10ft board into two pieces, where one piece is four times longer than the other piece, the carpenter must cut the board into one piece that is 2 ft long, and another piece that is 8 ft long.

Solving Problems with Two Variables

Example 1: A plane travels with the wind from Kansas City, MO, to Denver, CO, a distance of 600 miles in 2 hours. The return trip against the wind takes 3 hours. Find the speed of the plane with no wind. Also find the speed of the wind.

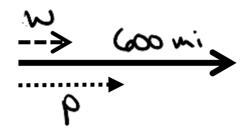
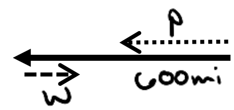
Step 1: Read and Identify

A plane travels with the wind from Kansas City, MO, to Denver, CO, a distance of 600 miles in 2 hours. The return trip against the wind takes 3 hours. Find the speed of the plane with no wind. Also find the speed of the wind.	
What information has been given?	The distance between the two locations is 600 miles. The plane makes the trip in 2 hours when traveling with the wind. The trip takes 3 hours traveling against the wind.
What information is missing?	The speed of travel on trip 1. The speed of travel on trip 2. The speed of the plane alone. The speed of the wind.
What is the question asking for?	The speed of the plane. The speed of the wind.
Identify any relevant formulas	Distance = speed x time traveling ⇨ Speed $= \frac{\text{distance}}{\text{time}}$

Step 2: Name and Label

Knowns	Unknowns	Goal
Distance 1 $= 600\text{ mi}$	Speed of Plane $= p$	$p = ?$
Distance 2 $= 600\text{ mi}$	Speed of Wind $= w$	$w = ?$
Time 1 $= 2\text{ hrs}$	Speed Trip 1 $= ?$	
Time 2 $= 3\text{ hrs}$	Speed Trip 2 $= ?$	

Step 3: Describe and Translate

Describe	Translate
Speed is equal to distance traveled divided by the amount of time traveling	$\text{Speed} = \dfrac{\text{distance}}{\text{time}}$
The speed of Trip 1 is equal to the distance traveled, 600 mi, divided by time, 2 hrs.	$\text{Speed 1} = \dfrac{600\ \text{mi}}{2\ \text{hrs}} = 300\ \text{mph}$
The speed of Trip 2 is equal to the distance traveled, 600 mi, divided by the time, 3 hrs.	$\text{Speed 2} = \dfrac{600\ \text{mi}}{3\ \text{hrs}} = 200\ \text{mph}$
Since both the wind and the plane are heading in the same direction during Trip 1, the speed of the wind is creating a boost to the speed of the plane. And since the wind and the plane are heading in opposite directions during Trip 2, the speed of the wind is going against the speed of the plane.	 
This means that the speed of Trip 1 is also equal to the combined speed of the plane and the wind	$\text{Speed 1} = \text{p} + \text{w}$
This means that the total speed of Trip 2 is the difference between the speed of the plane and the speed of the wind.	$\text{Speed 2} = \text{p} - \text{w}$

Step 4: Solve

Use elimination to solve:

$$\begin{aligned} p + w &= 300\ mph \\ +\quad p - w &= 200\ mph \\ \hline 2p &= 500\ mph \\ \frac{2p}{2} &= \frac{500\ mph}{2} \\ p &= 250\ mph \end{aligned} \quad \Rightarrow \quad \begin{aligned} 250 + w &= 300\ mph \\ -250 \quad & \quad -250 \\ w &= 50\ mph \end{aligned}$$

Knowns	Unknowns	Answers
Distance 1 $= 600$ mi	Speed of Plane $= p$	$= 250$ mph
Distance 2 $= 600$ mi	Speed of Wind $= w$	$= 50$ mph
Time 1 $= 2$ hrs	Speed Trip 1 $= 300$ mph	
Time 2 $= 3$ hrs	Speed Trip 2 $= 200$ mph	

Step 5: Describe Solution

Example 1 A plane travels with the wind from Kansas City, MO, to Denver, CO, a distance of 600 miles in 2 hours. The return trip against the wind takes 3 hours. Find the speed of the plane with no wind. Also find the speed of the wind.

Answer The speed of the plane in still air is 250 mph, the speed of the wind is 50 mph. During Trip 1 when the plane was traveling with the wind, the total speed was 300 mph, allowing the plane to make the 600-mile trip in 2 hours. During Trip 2, when the plane was traveling against the wind, the total speed was reduced to 200 mph, meaning the 600-mile trip took 3 hours

Solving Problems with Multiple Variables

Example 2: You decide to invest $5000 for 5 years split into two accounts. The first account pays 4% interest per year, compounded quarterly. The second pays 2% per year, compounded continuously. At the end of 5 years the combined value of the two accounts is $5,870.91, how much was initially invested in each account? Round the final answer to the nearest whole dollar amount.

Step 1: Read and Identify.

You decide to invest $5000 split into two accounts. The first account pays 4% interest per year, compounded quarterly. The second pays 2% per year, compounded continuously. At the end of 5 years the combined value of the two accounts $5,870.91, how much was initially invested in each account? Round the final answer to the nearest whole dollar amount.	
What information has been given?	Initial amount = $5,000 Final accumulated amount = $5,870.91 The sum of the initial investment in account A and account B is equal to $5,000. The sum of the accumulated values in account A and account B after 5 years is equal to $5,870.91 Account A interest = 4% per year, compounded quarterly, so 4 times a year. Account B interest = 2% per year, compounded continuously Time invested = 5 years

What information is missing?	Amount invested in account A Amount invested in account B Accumulated value of account A Accumulated value of account B
What is the question asking for?	Amount invested in account A Amount invested in account B
Identify any relevant formulas	Monthly compounding: $A = P\left(1 + \frac{r}{n}\right)^{nt}$ n = number of times per year r = interest rate t = time Continuous compounding: $A = Pe^{rt}$

Step 2: Name and Label information.

Knowns	Unknowns	Goal
Initial $= \$5{,}000$	Initial investment A $= P_A$	P_A
Final $= \$5{,}870.91$	Initial investment B $= P_B$	P_B
Time $= 5$ years	Accumulated A $= A_A$	
Rate A $= 4\% = 0.04$	Accumulated B $= A_B$	
Rate B $= 2\% = 0.02$		
$\text{n} = 4$		

Step 3: Describe and Translate

Describe	Translate
The initial investment of $5,000 dollars is split into account A and account B.	$P_A + P_B = \$5{,}000$
The accumulated amount of $5,870.91 dollars is the sum of the accumulated amount in account A and account B.	$A_A + A_B = \$5{,}870.91$
The monthly compounding formula, $A = P\left(1 + \frac{r}{n}\right)^{nt}$, tells us the accumulated value in account A, with an initial investment, P_A, a rate of 4%, compounded 4 times a year, for 5 years.	$A_A = P_A\left(1 + \frac{0.04}{4}\right)^{4 \cdot 5}$ $= P_A(1.01)^{20}$
The continuous compounding formula, $A = Pe^{rt}$, tells us the accumulated value in account B with an initial investment, P_B, a rate of 2%, compounded continuously for 5 years.	$A_B = P_B e^{0.02 \cdot 5}$ $= P_B e^{0.1}$

Step 4: Solve the equation.

We can use substitution to eliminate the unknown variables, A_A and A_B, and replace them with the goal variables, P_A and P_B.

(1)	$A_A + A_B$	=	$\$5,870.91$	Substitute
(1.1)	$P_A(1.01)^{20} + P_B(e^{0.1})$	=	$\$5,870.91$	
(2)	$P_A + P_B$	=	$\$5000$	Solve for P_B
(2.1)	P_B	=	$\$5000 - P_A$	Substitute into equation (1.1)
(3)	$P_A(1.01)^{20} + (\$5000 - P_A)e^{0.1}$	=	$\$5,870.91$	Distribute and combine like-terms
(3.1)	$P_A[(1.01)^{20} - e^{0.1}]$	=	$5870.91 - 5000e^{0.1}$	Divide
(3.2)	P_A	=	$\frac{5870.91 - 5000e^{0.1}}{[(1.01)^{20} - e^{0.1}]}$	
(3.3)	P_A	=	$\$2,999.98$	Substitute into equation (2)
(4)	$2999.98 + P_B$	=	$\$5000$	Solve
(4.1)	P_B	=	$\$2000.02$	

Knowns	Unknowns	Goal
Initial = $\$5,000$	Initial investment A = P_A	$P_A = \$3000$
Final = $\$5,870.91$	Initial investment B = P_B	$P_B = \$2000$
Time = $5\ years$	Accumulated A = A_A	
Rate A = $4\% = 0.04$	Accumulated B = A_B	
Rate B = $5\% = 0.02$		
n = 4		

Step 5: <u>Describe</u>

Example 2: You decide to invest $5000 for 5 years split into two accounts. The first account pays 4% interest per year, compounded quarterly. The second pays 2% per year, compounded continuously. At the end of 5 years the combined value of the two accounts is $5,870.91, how much was initially invested in each account? Round the final answer to the nearest whole dollar amount.

Answer: The initial investment placed in the 4% quarterly compounded account is $3,000. The initial investment placed in the 2% continuously compounded account is $2,000. After 5 years of investing, the combined value of both accounts is $5,870.91.