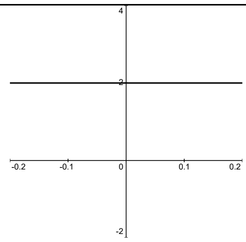
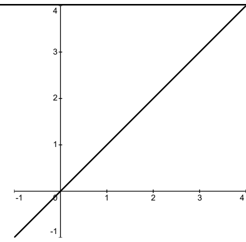
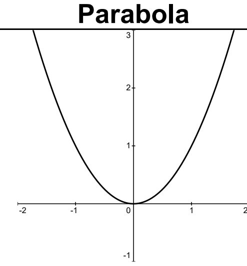
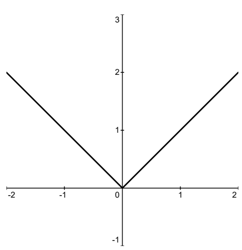
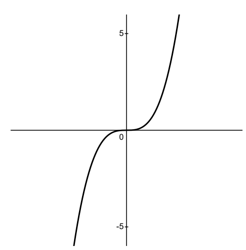
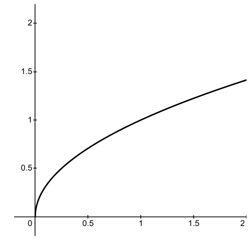
Graphing: Functions and Transformations

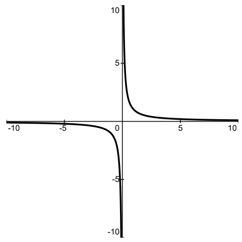
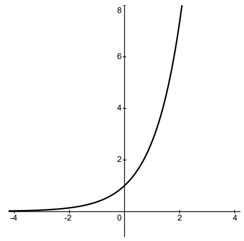
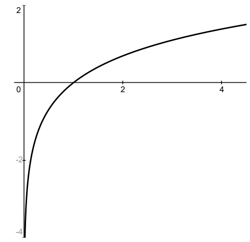
For the purposes of this handout, it is important to clearly define "Function".

<u>Function</u>: A **relation** between two corresponding sets, **Domain** and **Range**. One being a set of all potential **inputs** (x-values), each one having their own unique **output** (y-value), respectively. If there is more than one **output** for a unique **input**, it is not a **function**.

*__Note__: A **vertical line test** can determine if a curve is a function. If the vertical line crosses through the graphed curve twice, it is not a function.

Parent Function Graphs

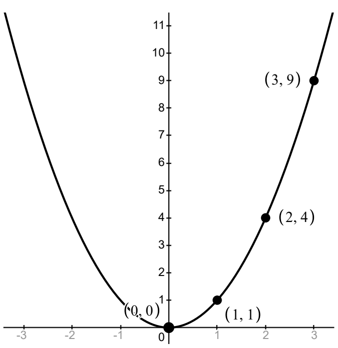
Constant	**Linear**	**Parabola**
		
$f(x) = c$ Ex: $c = 2$	$f(x) = x$	$f(x) = x^2$
Domain: $(-\infty, \infty)$ **Range**: y $= c$ The function progresses along the *x-axis*, $f(x) = 2$, for every *x-value*. This follows for any constant **c** Constant, neither increasing or decreasing toward infinity.	**Domain**: $(-\infty, \infty)$ **Range**: $(-\infty, \infty)$ For every value of x there is an identical $f(x)$ value. Example: When $x = 1$, $f(x) = 1$, and so on.	**Domain**: $(-\infty, \infty)$ **Range**: $(0, \infty)$ Decreasing when $-\infty \leq f(x) \leq 0$ Increasing when $0 \leq f(x) \leq \infty$
Absolute	**Cubic**	**Square Root**
		
$f(x) = \lvert x \rvert$	$f(x) = x^3$	$f(x) = \sqrt{x}$
Domain: $(-\infty, \infty)$ **Range**: $(0, \infty)$ Decreasing when $-\infty \leq f(x) \leq 0$ Increasing when $0 \leq f(x) \leq \infty$	**Domain**: $(-\infty, \infty)$ **Range**: $(-\infty, \infty)$ Always Increasing to the right, and always decreasing to the left, from origin.	**Domain**: $[0, \infty)$ **Range**: $[0, \infty)$ Always Increasing from 0 to infinity.

Inverse	**Exponential**	**Logarithmic**
		
$f(x) = \frac{1}{x}$	$f(x) = e^x$	$f(x) = \ln x$
Domain: $(-\infty, 0) \cup (0, \infty)$ **Range**: $(-\infty, 0) \cup (0, \infty)$ Decreases as x approaches 0 from the left-side of the *y-axis*. Decreases as x moves away from 0 on the right-side of the *y-axis.*	**Domain**: $(-\infty, \infty)$ **Range**: $(0, \infty)$ Increases as x moves from $-\infty$ to ∞ This function expresses "*Exponential Growth*".	**Domain**: $(0, \infty)$ **Range:** $(-\infty, \infty)$ Increases as x moves from 0 to ∞ Notice, this is the inverse of $f(x) = e^x$

Transformations

These transformations are applicable to all the **Parent Functions** shown above. Notice the coordinates in the following **Parent Function**, $f(x) = x^2$.

<u>Parent Function</u>

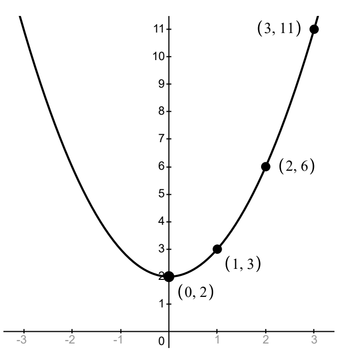


x	y
0	*0*
1	*1*
2	*4*
3	*9*
-1	*1*
-2	*4*

Notice the *vertex* is located at **0** for both *x* and *y values*.

Recall, that both the positive and negative of the same *x-value* share the same *y-value*.

<u>Transformation Function</u>

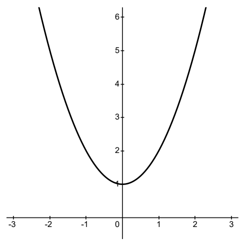
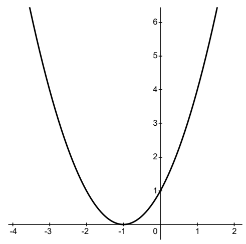
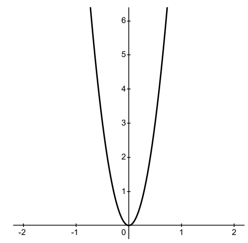
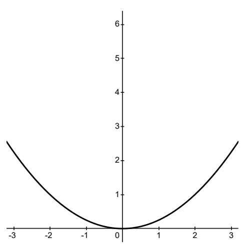
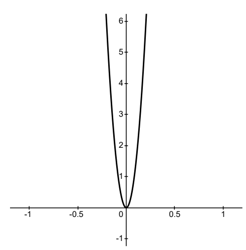
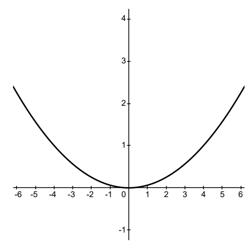


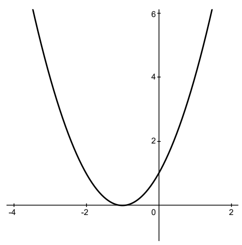
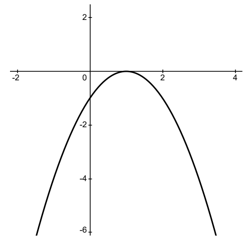
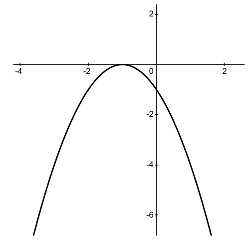
x	Y
0	*2*
1	*3*
2	*6*
3	*11*
-1	*3*
-2	*6*

Example A: $f(x) = x^2 + 2$

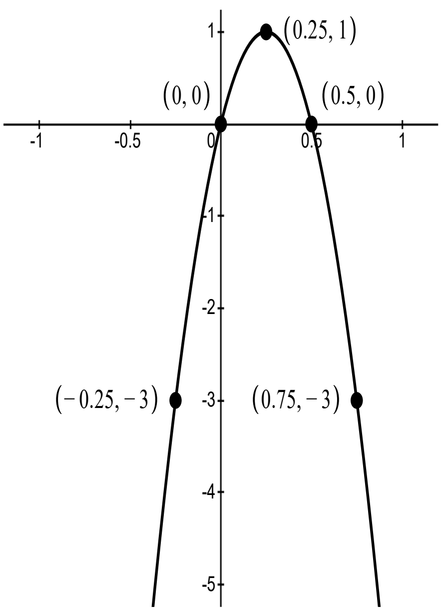
Notice the *vertex* is now shifted up by **2**, on the *y axis*, yet still holds the **0** position on the *x axis*.

Thus, this pattern follows for all other *y* values, their output being increased by a sum of **2**.

Vertical Shift	Horizontal Shift	Vertical Stretch
		
General Form: $x^2 \pm k$ Example: $f(x) = x^2 + 1$	General Form: $(x \pm h)^2 + k$ Example: $f(x) = (x + 1)^2$	General Form: $cf(x), c > 1$ Example: $f(x) = 12(x^2)$
This example shows a standard **Parent Graph** parabola shifted up by a value of **1**, for all *y-values*. This transformation, under the general form, is applicable to any parent graph. The function can shift *upward* by *addition*, and *downward* by *subtraction*.	When there is addition inside the parentheses, the curve shifts "*h-units*" the left. When there is subtraction inside the function the curve shifts "*h-units*" to the right. We can shift with respect to any numerical value given. If a $+2$ is in the "h" place, the graph shifts two *x-values* to the left.	Notice the coefficient is outside the parentheses of the function. The coefficient is directly proportional to the function *f(x)*. In this example, every *y-coordinate* is multiplied by a factor of **12**.
Vertical Compression	**Horizontal Compression**	**Horizontal Stretch**
		
General Form: $cf(x), 0 < c < 1$ Example: $f(x) = \frac{1}{4}(x^2)$	General Form: $f(cx), c > 1$ Example: $(12x)^2$	General Form: $f(cx), 0 < c < 1$ Example: $f(x) = \left(\frac{1}{4}x\right)^2$
The *coefficient* is outside the parentheses of the function. This implies that all outputs of *y*, that is *f(x)* are multiplied by a factor of $\mathbf{\frac{1}{4}}$**.**	Here the coefficient is inside the parentheses of the function. Here, the value *c* compresses the curve horizontally, making a steeper slope by the reciprocal of the *c*-value, in this case by a factor of $\mathbf{\frac{1}{12}}$**.**	Notice the coefficient is inside the parentheses of the function, as previously shown. In this case, the value *c* horizontally stretches the function by the reciprocal of the *c*-value, in this case by a factor of 4.

Y-Axis Reflection	X-Axis Reflection	X,Y Reflection
		
$f(x) = (-x-1)^2$	$f(x) = -(x-1)^2$	$f(x) = -(-x-1)^2$
Notice, this shift is reflected over the *y-axis*, as if reading: $f(x) = (x+1)^2$ When the negative is inside the function, the function reflects about the *y-axis*. Due to the reflection, all the *x-values* change their sign from positive to negative.	Notice, this same function now reflects about the *x-axis*. This is due to the negative sign being on the outside of the function. Again, consider the function: $f(x) = (x-1)^2$ Due to the reflection, all the *y-values* change their sign from positive to negative.	Consider the previous two *transformations*, applying both *transformations* to this function allows for two reflections. This is a function that reflects both about the *y-axis* and about the *x-axis*. Thus, all values of *x* and *y* have changed signs.

Transformation Function



X	y
0	*0*
$\frac{1}{2}$	*0*
$\frac{3}{4}$	*-3*
$-\frac{1}{4}$	*-3*
$\frac{1}{4}$	*1*

<u>Example</u>

$f(x) = -(-4x+1)^2 - 1$

In this unique **Transformation**, there is a vertical shift, horizontal shift, horizontal stretch, and reflection over both axes.
The function moves down by **1**, left by **1**, stretches horizontally by a factor of **4**, then reflects over the *x-axis* and *y-axis* from **the Parent Function's** original position.
As a result of the combined reflections and prior shifts, the vertex is relocated upward by **1** unit on the *y-axis* and rightward by $\frac{1}{4}$ unit on the *x-axis*.

All graphs are freely generated via 'Desmos Graphing Calculator.' At desmos.com/graphing