

**MATH 250 Review for Test 2**  
**(Revised April 8, 2009)**

A. (1st Order Linear DE's) Find the general solution:

- 1)  $y^2 dx - x(2x + 3y)dy = 0$       Ans:  $y^2(x + y) = Cx$   
2)  $y(x + 3y)dx + x^2 dy = 0$       Ans:  $yx^2 = C(3y + 2x)$   
3)  $6x^2 dy - y(x + 2y^3)dx = 0$       Ans:  $(x - 2y^3)^2 = Cxy^6$   
4)  $(3x^2 y + 2xy^2)dx + (4x^3 + 5x^2 y)dy = 0$   
    Ans:  $x^3 y^4 + x^2 y^5 = C$

B. Find the general solution:

- 1)  $(D^2 - D - 2)y = 0$       4)  $(D^2 + 2D + 4)y = 0$   
2)  $\frac{d^2 y}{dx^2} + 5\frac{dy}{dx} + 6y = 0$       5)  $(D^2 - 2D - 5)y = 0$   
3)  $\frac{d^2 y}{dx^2} + 10\frac{dy}{dx} + 25y = 0$       6)  $(D^2 + 16)y = \sin 5x$

C. Find the specific solution:

- 1)  $\frac{d^2 y}{dx^2} + \frac{dy}{dx} - 12y = 0$  where  $y(0) = 1$  and  $y'(0) = 0$ .  
2)  $\frac{d^2 y}{dx^2} - 2\frac{dy}{dx} + y = 0$  where  $y(0) = 0$  and  $y'(0) = 2$ .  
3)  $(D^2 - 1)y = e^{2x}$  where  $y(0) = 0$  and  $y'(0) = 0$ .

D. Find the general solution:

- 1)  $(D^2 + 25)y = \sin 5x$       4)  $\frac{d^2 y}{dx^2} - 4\frac{dy}{dx} = 10$   
2)  $(D^2 - 9)y = e^{3x}$       5)  $(D^2 + 4D + 3)y = \sin 3x$   
3)  $(D^2 + 3D + 2)y = 1 + x$

E. Prove that the functions  $\cos 3x$  and  $\sin 3x$  are linearly independent.

F. Show that  $D(xD - 1) = xD^2$ .

G. Solve  $(D^2 + 2D + 5)y = 8e^{-x}$  such that  $y(0) = 0$  and  $y'(0) = 8$ .  
Ans:  $y = 4e^{-x} \sin 2x - 2e^{-x} \cos 2x + 2e^{-x}$

H. Find the general solution to  $(D^2 - 2D - 3)y = 3x^2 - 5$ .  
Ans:  $y = Ae^{3x} + Be^{-x} - x^2 + \frac{4}{3}x + \frac{1}{9}$

I. Solve  $(D^2 + 1)y = 12 \cos^2 x$ . Hint:  $\cos^2 x = \frac{1 + \cos 2x}{2}$   
Ans:  $y = A \sin x + B \cos x + 6 - 2 \cos 2x$

J. A nitric acid solution flows at a constant rate of 8 L/min into a tank that initially held 180 L of 6% nitric acid solution. The solution inside the tank is kept well stirred and flows out of the tank at a rate of 10 L/min. If the solution entering the tank is 10% nitric acid, find the DE for  $x(t)$  where  $x(t)$  is the volume of nitric acid after  $t$  minutes and find  $x(0)$ . What is the solution to this DE?