Fundamentals of Differential Equations
by Nagle, Saff, and Snider (7th edition)

Section 7.3 (p. 391)- Properties of Laplace Transforms

3. To find $L(e^{-t}cos\,3t + e^{6t} - 1)$

   note that $L(e^{-t}cos\,3t) = f(s+1)$ where

   $f(s) = L(cos\,3t) = \dfrac{s}{s^2+9}$.

   $$L(e^{-t}cos\,3t + e^{6t} - 1) = \frac{s+1}{(s+1)^2+9} + \frac{1}{s-6} - \frac{1}{s}$$

5. To find $L(2t^2e^{-t} - t + cos\,4t)$

   note that $L(2t^2e^{-t}) = f(s+1)$ where

   $f(s) = L(2t^2) = 2\dfrac{2}{s^3} = \dfrac{4}{s^3}$.

   $$L(2t^2e^{-t} - t + cos\,4t) = \frac{4}{(s+1)^3} - \frac{1}{s^2} + \frac{s}{s^2+16}$$

7. $L((t-1)^4) = L(t^4 - 4t^3 + 6t^2 - 4t + 1) =$

   $$\frac{4!}{s^5} - 4\frac{3!}{s^4} + 6\frac{2!}{s^3} - \frac{4}{s^2} + \frac{1}{s} = \frac{24}{s^5} - \frac{24}{s^4} - \frac{4}{s^2} + \frac{1}{s}$$

11. To find $L(cosh\,bt)$ use

    $L(F'') = s^2L(F) - sF(0) - F'(0)$ where

    $F(t) = cosh\,bt,\ F'(t) = b\,sinh\,bt,\ F''(t) = b^2cosh\,bt$

    $L(b^2cosh\,bt) = s^2L(cosh\,bt) - s\cdot 1 - 0 \quad \Rightarrow$

    $s = L(cosh\,bt)[s^2 - b^2] \quad \Rightarrow \quad L(cosh\,bt) = \dfrac{s}{s^2-b^2}$.

13. $sin^2t = \dfrac{1-cos\,2t}{2} = \dfrac{1}{2} - \dfrac{1}{2}cos\,2t$

    $$L(sin^2t) = \frac{1}{2s} - \frac{1}{2}\frac{s}{s^2+4} = \frac{1}{2s} - \frac{2}{2s^2+8}$$

21. Since $L(e^{at}F(t)) = f(s-a)$ where $f(s) = L(F(t))$ and $L(cos\,bt) = \dfrac{s}{s^2+b^2}$,

$$L(e^{at}cos\,bt) = \frac{s-a}{(s-a^2)+b^2}\,.$$