**Fundamentals of Differential Equations**
**by Nagle, Saff, and Snider (7th edition)**

Section 4.2 (p. 176)- Homogeneous Linear Equations: the General Solution

5. Find a general solution to the DE $y'' + 8y' + 16y = 0.$

$m^2 + 8m + 16 = 0 \;\; \Rightarrow \;\; (m+4)^2 = 0 \;\; \Rightarrow \;\; m = -4$ is a root of mult. 2 $\;\; \Rightarrow \;\;$ the general solution is

$$y = C_1 e^{-4t} + C_2 x e^{-4t}.$$

15. Solve the initial value problem
$y'' - 4y' - 5y = 0; \; y(-1) = 3, \; y'(-1) = 9$

$m^2 - 4m - 5 = 0 \;\; \Rightarrow (m+1)(m-5) = 0$
$\Rightarrow \;\; m = -1, 5 \;\; \Rightarrow$ the general solution is
$y = ae^{-t} + be^{5t}.$
$y' = -ae^{-t} + 5be^{5t}$
$y(-1) = 3 \;\; \Rightarrow \quad ae + be^{-5} = 3$
$y'(-1) = 9 \;\; \Rightarrow \quad -ae + 5be^{-5} = 9$
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$6be^{-5} = 12 \;\; \Rightarrow \;\; b = 2e^5$

$ae + 2e^5 e^{-5} = 3 \;\; \Rightarrow \;\; ae = 1$
$\Rightarrow \;\; a = e^{-1}.$

The specific solution is $y = e^{-1}e^{-t} + 2e^5 e^{5t}$
$\Rightarrow \;\; y = e^{-1-t} + 2e^{5t+5}.$

19. Solve the initial value problem
$y'' + 2y' + y = 0; \; y(0) = 1, \; y'(0) = -3$

$m^2 + 2m + 1 = 0 \;\; \Rightarrow \;\; (m+1)^2 = 0 \;\; \Rightarrow \;\; m = -1$
is a root of mult. 2.

The general solution is
$y = ae^{-t} + bxe^{-t}$
$y' = -ae^{-t} + be^{-t} - bte^{-t}$

$y(0) = a + 0 = 1 \quad \Rightarrow \quad a = 1\,.$
$y'(0) = -a + b = -3 \quad \Rightarrow \quad -1 + b = -3 \quad \Rightarrow \quad b = -2$
The specific solution is $y = e^{-t} - 2te^{-t}\,.$

29. Use Def. 1 to determine whether the functions $f(t) = te^{2t}$ and $g(t) = e^{2t}$ are linearly independent on the interval $(0, 1)$.

$f(t)$ and $g(t)$ are linearly independent if there is no constant $k$ such that $kf(t) = g(t)$ or more simply
$g(t) = kf(t).$
Say $f(t) = kg(t) \quad \Rightarrow \quad te^{2t} = ke^{2t}$ for all $t \in (0, 1).$
Take $t = 1/2.$ Then $\frac{1}{2}e^{1} = ke^{1} \quad \Rightarrow \quad k = \frac{1}{2}\,.$
But then take $t = 1/4 \;\Rightarrow\; \frac{1}{4}e^{t/2} = ke^{t/2} \quad \Rightarrow$
$k$ would have to be $\frac{1}{4}\,.$
Therefore no such constant exists.

31. Use Def. 1 to determine whether the functions $f(t) = tan^2 t - sec^2 t$ and $g(t) = 3$ are linearly independent on the interval $(0, 1)$.

$1 + tan^2 t = sec^2 t \quad \Rightarrow \quad tan^2 t - sec^2 t = -1\,.$
Therefore $-3(tan^2 t - sec^2 t) = -3(-1) = 3$
and $-3f(t) = g(t) \quad \Rightarrow \quad f(t)$ and $g(t)$ are linearly dependent.

39. Find 3 linearly independent solutions of the 3rd order linear DE

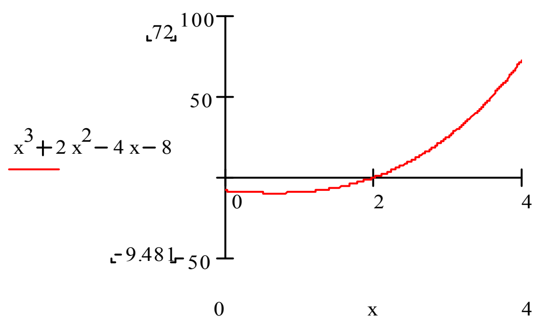
$$\frac{d^3z}{dz^3} + 2\frac{d^2z}{dt} - 4\frac{dz}{dt} - 8z = 0$$

and wire a general solutions as a linear combination of these.

$m^3 + 2m^2 - 4m - 8 = 0$

We must find one solution. The integer solutions can be $\pm 1$, 2, 4, 8.

Looking at the graph of $f(x) = x^3 + 2x^2 - 4x - 8$ we can see that 2 is a solution.




Using synthetic division and 2 we get

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1    2   -4   -8   \ 2
     2    8    8
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1   4     4     0
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This means 2 is a solution and $x - 2$ is a factor.

$m^3 + 2m^2 - 4m - 8 = (m-2)(m^2 + 4m + 4) =$

$(m-2)(m+2)^2 \quad \Rightarrow$

$m = -2$ is a root of mult. 2.

3 linearly independent solutions are $e^{2t}$, $e^{-2t}$, and $te^{-2t}$.

The general solution is $C_1e^{2t} + C_2e^{-2t} + C_3te^{-2t}$.