Fundamentals of Differential Equations
by Nagle, Saff, and Snider (7th edition)

Section 1.4 (p. 28)- The Approximation Method of Euler

1. $dy/dx = y(2-y),\ y(0) = 3$
Estimate y at the pts where $x = 0.1,\ 0.2,\ 0.3,\ 0.4,$ and 0.5.
using $h = 0.1$.
New y $\approx$ old y + slope at old pt $\times\ \Delta x$

$$3 + 3\,(2-3)\,(.1) = 2.7$$

$$2.7 + 2.7\,(2-2.7)\,(.1) = 2.511$$

$$2.511 + 2.511\,(2-2.511)\,(.1) = 2.383$$

$$2.383 + 2.383\,(2-2.383)\,(.1) = 2.292$$

$y(.1) \approx 2.7,\ y(.2) \approx 2.511,\ y(.3) \approx 2.383,\ y(.4) \approx 2.292$
Finally,
$y(.5) \approx 2.292 + (2.292)(2-2.292)(.1) = 2.225$.

3. $\dfrac{dy}{dx} = \dfrac{x}{y},\ y(0) = -1.$
Estimate y at the pts where $x = 0.1,\ 0.2,\ 0.3,\ 0.4,$ and 0.5.
using $h = 0.1$.
New y = old y + slope at old pt $\times\ \Delta x$

$$y\,(at\ x = .1) \approx -1 + \frac{0}{-1}(.1) = -1.000$$

$$y(at\ x = .2) \approx -1.000 + \frac{0.1}{-1.000}(.1) = -1.010$$

$$y(at\ x = .3) \approx -1.010 + \frac{0.2}{-1.010}(.1) = -1.030$$

$$y(at\ x = .4) \approx -1.030 + \frac{0.3}{-1.030}(.1) = -1.059$$

$$y(at\ x = .5\,) \approx -1.059 + \frac{0.4}{-1.059}(.1) = -1.097$$

5. $y' = \frac{1}{x}\left(y^2 + y\right),\ y(1) = 1$

Estimate y at the pts where $x = 1.2,\ 1.4,\ 1.6$ using $h = 0.2$

$$1 + \frac{1^2 + 1}{1} \cdot 0.2 = 1.4$$

$$1.4 + \frac{1.4^2 + 1.4}{1.2} \cdot 0.2 = 1.96$$

$$1.96 + \frac{1.96^2 + 1.96}{1.4} \cdot .2 = 2.789$$

$y(1.2) = 1.4,\ y(1.4) = 1.96,\ \ y(1.6) = 2.789$