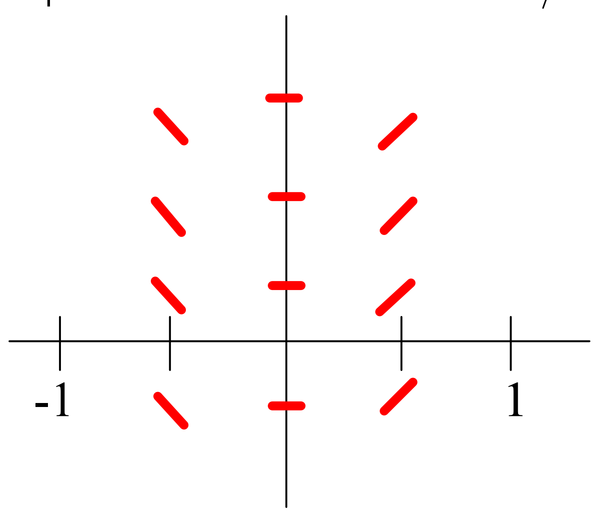
Fundamentals of Differential Equations
by Nagle, Saff, and Snider (7th edition)

Section 1.3 (p. 22)- Direction Fields

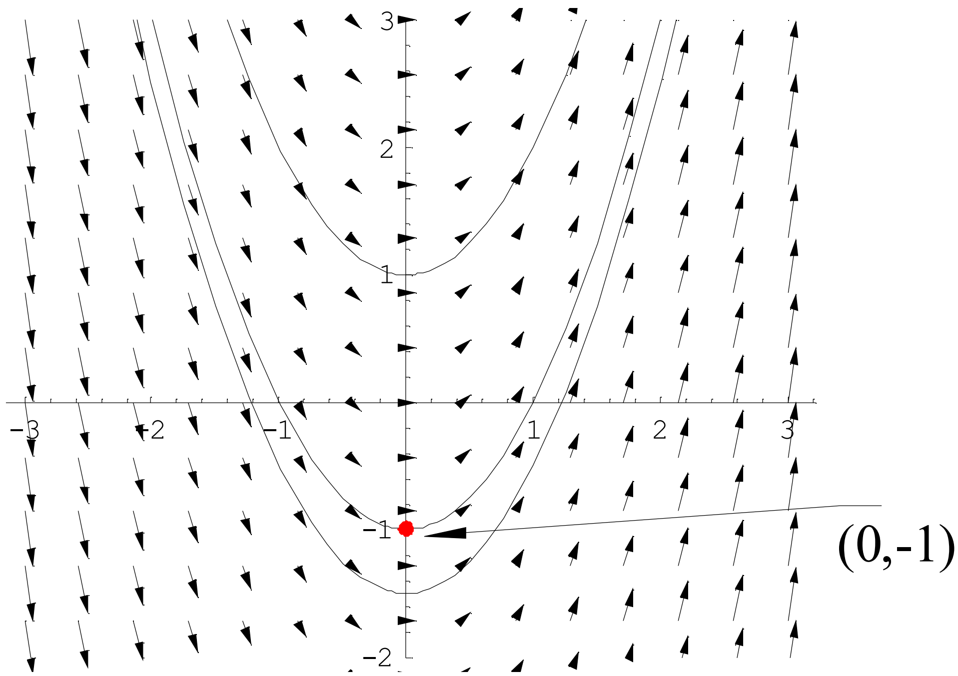
Draw the iscolines with their direction markers and sketch several solutions including the curve satisfying the initial conditions.

13. $\dfrac{dy}{dx} = 2x,\ y(0) = -1$

$m = 0 \quad \Rightarrow \quad 2x = 0 \quad \Rightarrow \quad x = 0$ (y-axis). Put hash marks of slope 0 (horiz. hash marks) on y-axis.
$m = 1 \quad \Rightarrow \quad 2x = 1 \quad \Rightarrow \quad x = \dfrac{1}{2}$. Put hash marks of slope 1 on vertical line $x = 1$.
$m = -1 \quad \Rightarrow \quad 2x = -1 \quad \Rightarrow \quad x = -1/2$. Put hash marks of slope -1 on vertical line $x = -1/2$.



The solution where $y(0) = -1$ is shown below, as well as 2 other solutions.

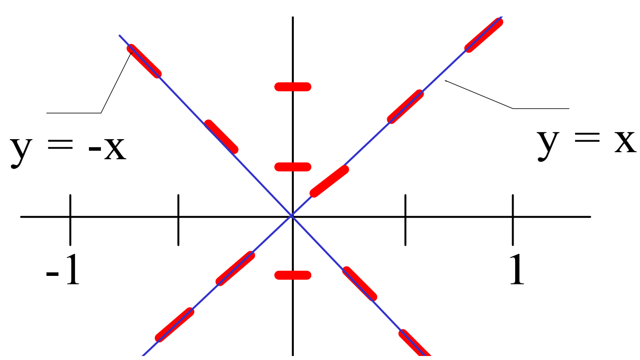


14. $\dfrac{dy}{dx} = \dfrac{x}{y},\ y(0) = -1$

$m = 0 \;\Rightarrow\; \dfrac{x}{y} = 0 \;\Rightarrow\; x = 0$. Put horiz. hash marks on line $x = 0$ (the y-axis).

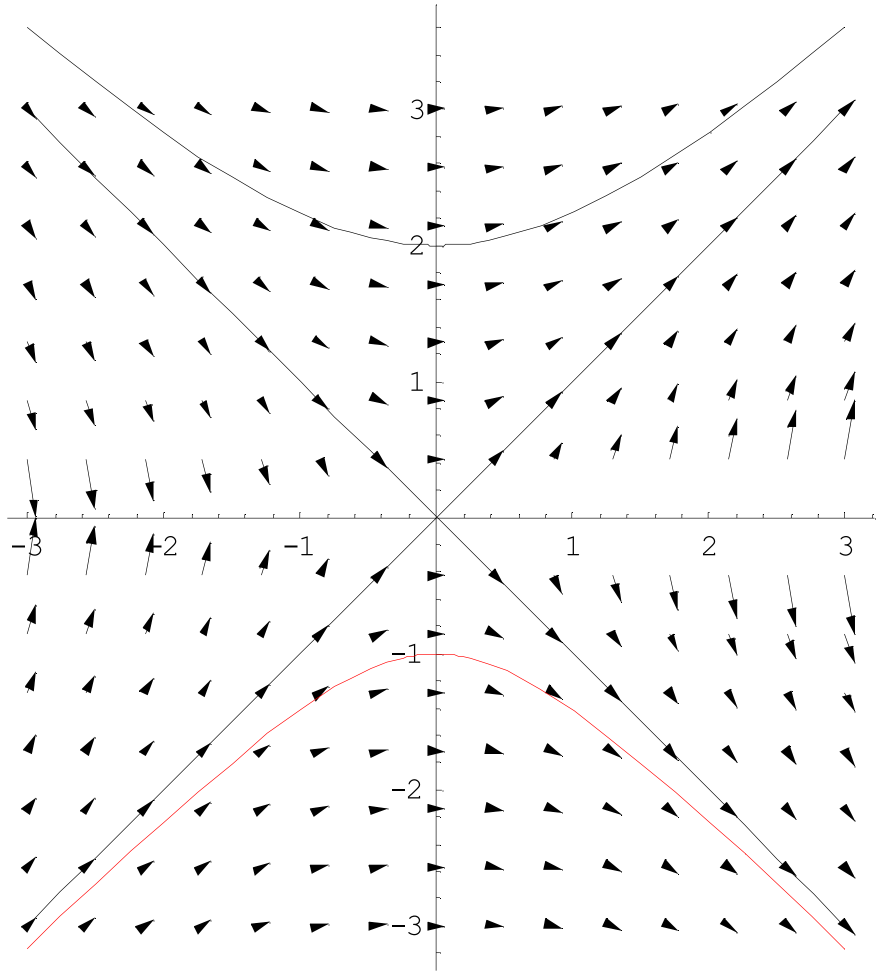
$m = 1 \;\Rightarrow\; \dfrac{x}{y} = 1 \;\Rightarrow\; x = y$. Put hash marks of slope 1 on the oblique line $y = x$.

$m = -1 \;\Rightarrow\; \dfrac{x}{y} = -1 \;\Rightarrow\; y = -x$. Put hash marks of slope -1 on the oblique line $y = -x$. See the graph below.



The solution curve where $y(0) = -1$ is shown below in red, along with 3 other solutions. Note that $y = x$ and

$y = -x$ are solutions.



15. $\dfrac{dy}{dx} = 2x^2 - y,\ y(0) = 0$

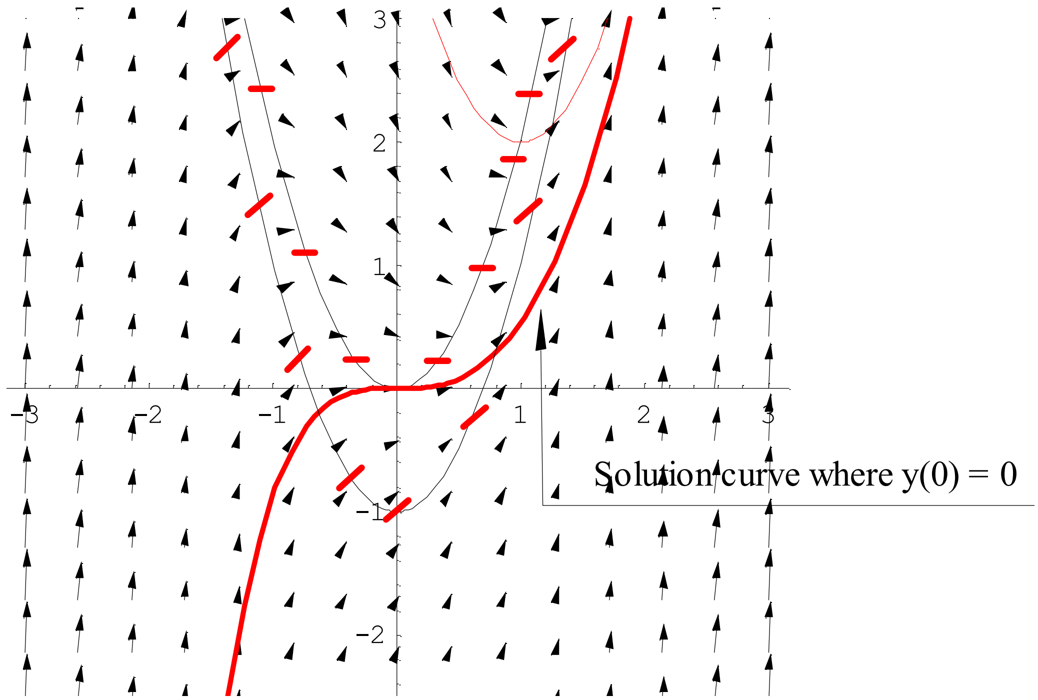
$m = 0 \Rightarrow 2x^2 - y = 0 \Rightarrow y = 2x^2.$

$m = 1 \Rightarrow 2x^2 - y = 1 \Rightarrow y = 2x^2 - 1$

$m = -1 \Rightarrow 2x^2 - y = -1 \Rightarrow y = 2x^2 + 1$

The iscoclines are parabolas of the form $y = 2x^2 + k$ where $m = -k$.

Shown below are 2 iscoclines (in black) where $m = 0$ and $m = 1$. The solution curve where $y(0) = 0$ is shown in bold red, as well as one other solution curve (the parabola $y = 2x^2 - 4x + 4$) also in red. You can check that $y = 2x^2 - 4x + 4$ is a solution.



Here is a plot of another solution curve, the function

$$y = -3e^{-x} + 2x^2 - 4x + 4.$$