# Detailed Solutions (Practice Test 3)

1. $L(5t^2 + 3t - 8) = \mathbf{10/s^3 + 3/s^2 - 8/s}$

2. $F(t) = t - \alpha(t-4)(t-5) = t - \alpha(t-4)(t-4-1)$

$$L(F(t)) = \mathbf{\frac{1}{s^2} - e^{-4s}\left(\frac{1}{s^2} - \frac{1}{s}\right)}$$

3. $D\,sinh\,kx = k\,cosh\,k\,x \qquad sinh\,0 = 0$

$D\,cosh\,kx = k\,sinh\,kx \qquad cosh\,0 = 1$

$L(F'') = s^2 L(F) - sF(0) - F'(0)$ where $F(t) = cosh\,kt$

$k^2 L(cosh\,kt) = s^2 L(cosh\,kt) - s \times 1 - 0$

$s = L(cosh\,kt)(s^2 - k^2)$

$$L(cosh\,kt) = \frac{s}{s^2 - k^2}$$

4. $$\frac{2s^2+1}{s(s+1)^2} = \frac{a}{s} + \frac{b}{s+1} + \frac{c}{(s+1)^2}$$

$2s^2 + 1 = a(s+1)^2 + bs(s+1) + cs$

If $s = 0$ then $1 = a$.

If $s = -1$ then $3 = -c \Rightarrow c = -3$

If $s = 1$ then $3 = 4a + 2b + c \Rightarrow 3 = 4 + 2b - 3 \Rightarrow 2b = 2$

$\Rightarrow b = 1.$

The inverse transform of $\frac{1}{s} + \frac{1}{s+1} + \frac{-3}{(s+1)^2}$ is $\mathbf{1 + e^{-t} - 3te^{-t}}$.

5. $$\frac{1}{s^3(s^2+1)} = \frac{a}{s} + \frac{b}{s^2} + \frac{c}{s^3} + \frac{ds+e}{(s^2+1)}$$

$1 = as^2(s^2+1) + bs(s^2+1) + c(s^2+1) + (ds+e)s^3$

$1 = as^4 + as^2 + bs^3 + bs + cs^2 + c + ds^4 + es^3$

$1 = (a+d)s^4 + (b+e)s^3 + (a+c)s^2 + bs + c$

Therefore $b = 0$, $c = 1$, $a + c = 0 \Rightarrow a = -1$, $b + e = 0 \Rightarrow e = 0$,

and $a + d = 0 \Rightarrow d = 1$.

$$\frac{1}{s^3(s^2+1)} = \frac{-1}{s} + \frac{0}{s^2} + \frac{1}{s^3} + \frac{1s+0}{(s^2+1)}$$

Note: $\frac{1}{s^3} = \frac{1}{2}\frac{2}{s^3}$. Use $L^{-1}\left(\frac{2}{s^3}\right) = t^2$.

The inverse transform is $-\mathbf{1}+\dfrac{\mathbf{1}}{\mathbf{2}}\boldsymbol{t^2}+\boldsymbol{\cos t}$.

6. $$\frac{1}{s^2+4s+4}=\frac{1}{(s+2)^2}$$
$$L^{-1}\left(\frac{1}{(s+2)^2}\right)=e^{-2t}L^{-1}(1/s^2)=\boldsymbol{e^{-2t}t}$$

7. $$\frac{2s-3}{s^2-4s+4+4}=\frac{2(s-2)+1}{(s-2)^2+4}=\frac{2(s-2)}{(s-2)^2+4}+\frac{1}{(s-2)^2+4}$$
$$L^{-1}\left(\frac{2s-3}{s^2-4s+8}\right)=\boldsymbol{2e^{2t}\cos(2t)+\frac{1}{2}e^{2t}\sin(2t)}$$

8. $y''+y'-2y=-4$ where $y(0)=2,\ y'(0)=3$

$$s^2f(s)-2s-3+sf(s)-2-2f(s)=-4/s$$

$$\left(s^2+s-2\right)f(s)=\frac{2s^2+5s-4}{s}$$

$$f(s)=\frac{2s^2+5s-4}{s(s+2)(s-1)}=\frac{2}{s}-\frac{1}{s+2}+\frac{1}{s-1}$$

$$\boldsymbol{y=2-e^{-2t}+e^{t}}$$

9. $12=\frac{1}{2}k \ \Rightarrow\ k=24$

$$\frac{12}{32}x''+24x=0$$

$x''+64x=0$ where $x(0)=1/3$ and $x'(0)=-2$

$x=a\sin 8t+b\cos 8t$ and $x'=8a\cos 8t-8b\sin 8t$
$x(0)=1/3\ \Rightarrow\ b=1/3$
$x'(0)=-2\ \Rightarrow\ -2=8a\ \Rightarrow\ a=-1/4$

Therefore the equation of motion is $\boldsymbol{x=-\frac{1}{4}\sin 8t+\frac{1}{3}\cos 8t}$

The amplitude is $A=\sqrt{1/16+1/9}=\sqrt{25/144}=5/12$ of a foot.

$x = \frac{5}{12} sin(8t + \phi)$ where $\phi$ is located in the quadrant containing the point $\left(-\frac{1}{4}, \frac{1}{3}\right)$, the 2nd quadrant, and $\phi = \pi - tan^{-1}\left(\frac{1/3}{1/4}\right) = 2.214$ .

So $\boldsymbol{x = \frac{5}{12} sin(8t + 2.214)}$

When the weight gets back to the equilibrium point,

$\frac{5}{12} sin(8t + 2.214) = 0 \quad \Rightarrow \quad 8t + 2.214 = 0$ or $\pi$ or $2\pi$, ...

Because we want to find the 1st positive value of t where this occurs, we want $8t + 2.214 = \pi \quad \Rightarrow \quad t = \frac{\pi - 2.214}{8} = \mathbf{0.116}$ seconds .

When the weight comes back down again, $x$ is at its maximum which implies that $8t + 2.214 = \pi/2,\ 5\pi/2,\ 9\pi/2,\ ...$ We want the 1st positive value of t where this occurs. Therefore

$8t + 2.214 = 5\pi/2 \quad \Rightarrow \quad t = \frac{5\pi/2 - 2.214}{8} = \mathbf{0.705}$ **seconds**.

10. $\frac{12}{32} x'' + .6x' + 24x = 0$

$x'' + 1.6x' + 64x = 0$ where $x(0) = 1/3$ and $x'(0) = -2$

$m = -.8 \pm i\sqrt{63.36} = -.8 \pm 7.96i$

$x = e^{-.8t}(a\, cos\, 8.0t \ + \ b\, sin\, 8.0t)$

$x(0) = 1/3 \quad \Rightarrow \quad a = 1/3$

$x' = -.8e^{-.8t}(a\, cos\, 8.0t \ + \ bsin\, 8.0\, t) + e^{-.8t}(-8.0\, a\, sin\, 8.0t + 8.0\, bcos\, t)$

$x'(0) = -2 \quad \Rightarrow \quad -2 = -.8\, a + 8b \quad \Rightarrow \quad -2 = -.8(1/3) + 8b$

$\Rightarrow \quad b = \frac{-2 + (.8)(.333)}{8} \approx -.22$

Ans: $\boldsymbol{x = e^{-.8t}(.33\, cos\, 8.0\, t \ - \ .22\, sin\, 8.0\, t)}$

11. $F(t) = t^2 - (t^2 - 3)\alpha(t - 2)$

$f(t - 2) = t^2 - 3 \quad \Rightarrow \quad f(t) = (t + 2)^2 - 3 = t^2 + 4t + 1$

$L(F(t)) = \frac{2}{s^3} - e^{-2s} L(f(t)) = \boldsymbol{\frac{2}{s^3} - e^{-2s}\left(\frac{2}{s^3} + \frac{4}{s^2} + \frac{1}{s}\right)}$

12. $G(t) = 1 - 1 \cdot \alpha(t - \pi/2)$

$$L(G(t)) = \frac{1}{s} - e^{-\frac{\pi}{2}s} \cdot \frac{1}{s}$$

$$s^2 f(s) - 0 - 1 + f(s) = \frac{1}{s} - e^{-\frac{\pi}{2}s} \cdot \frac{1}{s}$$

$$(s^2 + 1) f(s) = 1 + \frac{1}{s} - e^{-\frac{\pi}{2}s} \cdot \frac{1}{s}$$

$$f(s) = \frac{1}{s^2+1} + \frac{1}{s(s^2+1)} - e^{-\frac{\pi}{2}s}\left(\frac{1}{s(s^2+1)}\right)$$

$$f(s) = \frac{1}{s^2+1} + \frac{1}{s} - \frac{s}{s^2+1} - e^{-\frac{\pi}{2}s}\left(\frac{1}{s} - \frac{s}{s^2+1}\right)$$

$x(t) = 1 + sin\,t - cos\,t - \alpha(t - \pi/2)(1 - cos(t - \pi/2))$

Note that $\cos(t - \pi/2) = cos(\pi/2 - t) = sin\,t$

Ans: $\boldsymbol{x(t) = 1 + sin\,t - cos\,t - (1 - sin\,t)\,\alpha(t - \pi/2)}$