Solutions to Review for Test 1
(Revised March 11, 2009)

1. Substitute e^{mx} into the DE.

$$m^2e^{mx} - 5me^{mx} - 50e^{mx} = 0 \;\Rightarrow\; (m^2 - 5m - 50)e^{mx} = 0 \;\Rightarrow$$
$$m^2 - 5m - 50 = 0 \;\Rightarrow\; (m-10)(m+5) = 0$$
$$\Rightarrow \quad m = 10 \text{ and } -5\,.$$

2. Just differentiate both sides of $8x^2 - y^2 = C$ and you get the DE

$$16x - 2yy' = 0 \;\Rightarrow\; y' = \frac{8x}{y}\,.$$

3. Multiply the DE by the integrating factor xy and you get

$$xy\left[\left(\frac{3x}{y} + 2y\right)dx + \left(2x + \frac{3y}{x}\right)dy\right] = 0 \text{ or}$$

$(3x^2 + 2xy^2)dx + (2x^2y + 3y^2)dy = 0$ which is exact.

$$\frac{\partial}{\partial y}(3x^2 + 2xy^2) = 4xy \text{ and } \frac{\partial}{\partial x}(2x^2y + 3y^2) = 4xy$$

$$\int 3x^2dx = x^3 \text{ and } \int 2xy^2dx = x^2y^2 \text{ and } \int 3y^2dy = y^3$$

$F(x,y) = x^3 + x^2y^2 + y^3$ and the solution is $x^3 + x^2y^2 + y^3 = C$.

4. Separate the variables

$$\frac{dy}{dx} = e^{2x}e^{-y} \;\Rightarrow\; e^ydy = e^{2x}dx \;\Rightarrow\; e^y = e^{2x}/2 + C$$

or $2e^y = e^{2x} + C$.

5. Exact

$$(y^4 + 2xy)dx + (4xy^3 + x^2)dy = 0$$

$$\frac{\partial M}{\partial y} = 4y^3 + 2x, \quad \frac{\partial N}{\partial x} = 4y^3 + 2x$$ ← DE is exact

$f(x,y) = xy^4 + x^2y$ and the solution to the DE is

$$xy^4 + x^2y = C\ .$$

6. Separate the variables

$$2(1-x^2)\frac{dy}{dx} = y$$

$$\frac{2dy}{y} = \frac{dx}{1-x^2} \quad \Rightarrow \quad \frac{2dy}{y} + \frac{dx}{(x+1)(x-1)} = 0$$

$$\frac{1}{(x+1)(x-1)} = \frac{a}{x+1} + \frac{b}{x-1} \quad \Rightarrow$$

$$1 = a(x-1) + b(x+1)$$

$$x = -1 \quad \Rightarrow \quad 1 = -2a \quad \Rightarrow \quad a = -1/2$$

$$x = 1 \quad \Rightarrow \quad 1 = 2b \quad \Rightarrow \quad b = 1/2$$

$$\frac{2dy}{y} + \frac{-(1/2)dx}{x+1} + \frac{(1/2)dx}{x-1} = 0$$

$$2\,ln\,y + \frac{1}{2}ln(x-1) - \frac{1}{2}ln(x+1) = ln\,C$$

$$4\,ln\,y + ln(x-1) - ln(x+1) = ln\,C$$

$$\frac{y^4(x-1)}{x+1} = C \quad \Rightarrow \quad y^4 = \frac{C(x+1)}{x-1}\ .$$

7. Separate the variables

$$(x^2+1)dx + x^2y^2dy = 0 \quad \Rightarrow \quad \frac{x^2+1}{x^2}dx + y^2dy = 0$$

$$\Rightarrow \quad (1+x^{-2})dx + y^2dy = 0 \quad \Rightarrow \quad x - x^{-1} + y^3/3 = C$$

$$\Rightarrow \quad 3x^2 - 3 + xy^3 = Cx \ (or\,3Cx) \quad \Rightarrow$$

$$xy^3 = 3(1 + Cx - x^2)\ .$$

8. Exact

$$(x^3+y^3)dx + y^2(3x+ky)dy = 0$$

$$F(x,y) = x^4/4 + xy^3 + ky^4/4 \quad \Rightarrow \quad x^4 + 4xy^3 + ky^4 = C\ .$$

9. Linear

$y' + 2xy = x^3 \quad \Rightarrow \quad u(x) = e^{\int 2x dx} = e^{x^2}$

$ye^{x^2} = \int x^3 e^{x^2} dx$ ← do by integration by parts

$u = x^2;\ dv = xe^{x^2} dx$

$du = 2xdx;\ v = e^{x^2}/2$

$$\int x^3 e^{x^2} dx = \frac{x^2 e^{x^2}}{2} - \int 2x \frac{e^{x^2}}{2} dx = x^2 e^{x^2}/2 - e^{x^2}/2 + C$$

$ye^{x^2} = x^2 e^{x^2}/2 - e^{x^2}/2 + C \quad \Rightarrow \quad 2y = x^2 - 1 + Ce^{-x^2}$

Substitute $x = 1$ and $y = 2$

$4 = 1 - 1 + Ce^{-1} \quad \Rightarrow \quad C = 4\,e$

$2y = x^2 - 1 + 4e\,e^{-x^2} \quad \Rightarrow \quad 2y = x^2 - 1 + 4\,e^{1-x^2}.$

10. Separable

$$\frac{dy}{dx} - cos\,x = cos\,x\,tan^2 y$$

$$dy = cos\,x\left(1 + tan^2 y\right)dx \quad \Rightarrow \quad \frac{dy}{1 + tan^2 y} = cos\,x\,dx$$

$$\Rightarrow \quad \frac{dy}{sec^2 y} = cos\,x\,dx \quad \Rightarrow \quad cos^2 y\,dy = cos\,x\,dx \quad \Rightarrow$$

$$\frac{1 + cos\,2y}{2} dy = cos\,x\,dx \quad \Rightarrow \quad \frac{1}{2} y + \frac{sin\,2y}{4} = sin\,x\ + C$$

$\Rightarrow \quad 2y + sin\,2y = 4\,sin\,x + C \quad \Rightarrow$

$2y + 2\,sin\,y\,cos\,y = 4\,sin\,x + C \quad \Rightarrow$

$$y + sin\,y\,cos\,y = 2\,sinx\ + C.$$

11. Linear

$$cos\,x \frac{dy}{dx} = 1 - y - sin\,x \quad \Rightarrow \quad \frac{dy}{dx} + \frac{y}{cos\,x} = \frac{1 - sin\,x}{cos\,x} \quad \Rightarrow$$

$$\frac{dy}{dx} + sec\,x \cdot y = \frac{1 - sin\,x}{cox\,x} = sec\,x\ -\ tan\,x\,.$$

$$u(x) = e^{\int sec\,x\,dx} = e^{ln|sec\,x + tan\,x|} = sec\,x\ +\ tan\,x$$

$$y(sec\,x\ +\ tan\,x) = \int \left(sec^2 x - tan^2 x\right) dx = \int 1 dx = x + C$$

$$y\left(\frac{1}{cos\,x} + \frac{sin\,x}{cos\,x}\right) = x + C \quad \Rightarrow \quad y(1 + sin\,x) = (x + C)cos\,x\,.$$

12. Linear with r the dependent variable

$sin\,\theta\,\frac{dr}{d\theta} = -1 - 2r\,cos\,\theta$

$\frac{dr}{d\theta} + (2\,cot\,\theta)\,r = -csc\,\theta$

$u(\theta) = e^{\int 2\,cot\,\theta d\theta} = e^{ln(sin^2\theta)} = sin^2\theta$

$r\,sin^2\theta = -\int sin\,\theta\,d\theta = cos\,\theta + c$ and the solution is

$r sin^2\theta = cos\,\theta + C$.

13. Use $\frac{1}{M}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right)$ to get an integrating factor $u(y)$.

$(y^6 + 2xy^3)dx + (4xy^5 + x^2y^2)dy = 0$

$\frac{\partial M}{\partial y} = 6y^5 + 6xy^2$ and $\frac{\partial N}{\partial x} = 4y^5 + 2xy^2$

$\frac{1}{M}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = \frac{2y^5 + 4xy^2}{y^6 + 2xy^3} = \frac{2y^2(y^3 + 2x)}{y^3(y^3 + 2x)} = \frac{2}{y}$

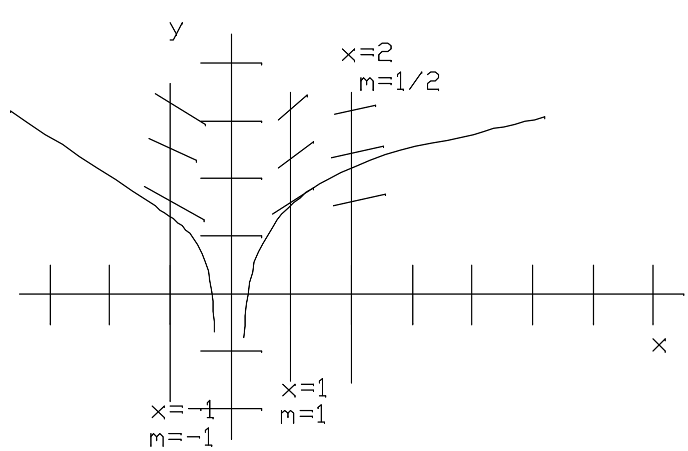
$u(y) = e^{-\int \frac{2}{y} dy} = e^{-2ln\,y} = e^{ln\,y^{-2}} = y^{-2}$

Mult. the DE by y^{-2} to get an exact DE.

$(y^4 + 2xy)dx + (4xy^3 + x^2)dy = 0$

$F(x, y) = xy^4 + x^2y$ and the solution is $xy^4 + x^2y = C$.

14. a) $\frac{dy}{dx} = \frac{1}{x}$



b) $\frac{dy}{dx} = 1/x$ and $y(1) = 1$

New y value = old y value + slope $\times$ .1
$y(1.1) \approx 1 + \frac{1}{1} \cdot 0.1 = 1.1$
$y(1.2) \approx 1.1 + \frac{1}{1.1} \cdot 0.1 \approx 1.19$
$y(1.3) \approx 1.19 + \dfrac{1}{1.2} \cdot 0.1 \approx 1.27$
Note that the solution is $y = 1 + ln\, x$.
$1 + ln(1.1) = 1.095$, $1 + ln(1.2) = 1.182$, and
$1 + ln(1.3) = 1.262$

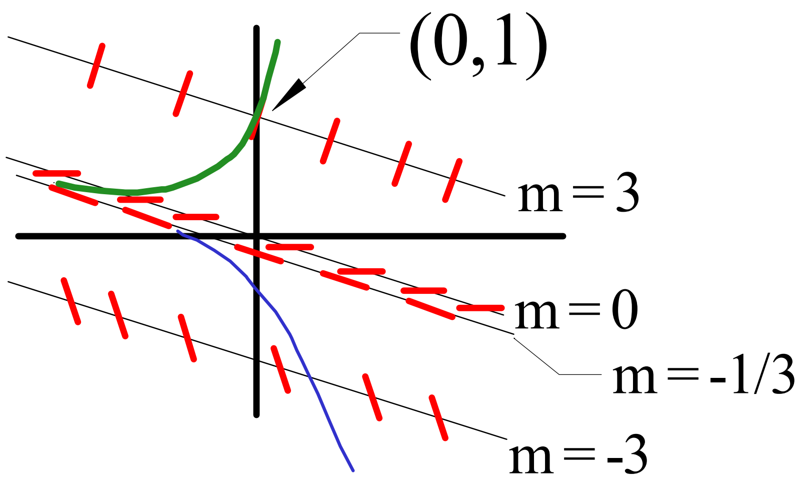
15. a) $\dfrac{dy}{dx} = 3y + x$

If $m = 0$, then $3y + x = 0 \Rightarrow y = -\dfrac{x}{3}$.

If $m = 3$, then $3y + x = 3 \Rightarrow y = -\dfrac{x}{3} + 1$.

In general, the isoclines have the form

$y = -\dfrac{x}{3} + \dfrac{m}{3}$ where $m/3$ is the y-intercept.



b) New y = old y +(old slope)Δx where slope $= 3y + x$.

$y(.002) \approx 1 + (3 + 0)(.002) = 1.006$

$y(.002) \approx 1.006 + (3[1.006] + .002)(.002) = 1.01204$

$y(.004) \approx 1.01204 + (3[1.01204] + .004)(.002) = 1.01819$