Algebra Formulas and Notes

Real Numbers and Their Properties

Order of Operations:

***P**lease*	parentheses, brackets, braces, fraction bars, radical signs, & absolute values
***E**xcuse*	exponents
***M**y **D**ear*	multiplication and division (left to right)
***A**unt **S**ally*	addition and subtraction (left to right)

Reciprocal of a is $1 \div a$ and of $a \div b$ is $b \div a$ $\quad a \div b = a \bullet 1/b$

Multiplication and division: The answer is positive if both numbers have the same sign.
The answer is negative if the numbers have different signs.

$0 \div a = 0 \qquad a \div 0 =$ undefined

To multiply fractions: divide the product of the numerators by the product of the denominators.

$(a \div b) \bullet (c \div d) = (a \bullet c) \div (b \bullet d) \qquad (a \div b) \div (c \div d) = (a \bullet d) \div (b \bullet c)$

If $a \div b = c \div d$, then $a \bullet d = b \bullet c$

To add fractions: replace each fraction with an equivalent one with a common denominator and then add the numerators together and use the common denominator.

$a \div c + b \div c = (a + b) \div c \qquad a \div c - b \div c = (a - b) \div c$

Properties:

Distributive	$a \bullet (b + c) = a \bullet b + a \bullet c$	
Inverse	$a + (-a) = 0$ and	$a \bullet 1 \div a = 1$
Identity	$a + 0 = a$ and	$a \bullet 1 = a$
Commutative	$a + b = b + a$ and	$a \bullet b = b \bullet a$
Associative	$a + (b + c) = (a + b) + c$ and	$a(bc) = (ab)c$
Multiplication of 0	$a \bullet 0 = 0$	

Multiplication property of equality: If $a = b$, then $a \bullet c = b \bullet c$ and $a \div c = b \div c$.

Addition property of equality: If $a = b$, then $a + c = b + c$ and $a - c = b - c$.

Inequalities:

If $a < b$, then for $c > 0$: $a \bullet c < b \bullet c$ and $a \div c < b \div c$.

If $a < b$, then for $c < 0$: $a \bullet c > b \bullet c$ and $a \div c > b \div c$.

If $a > b$, then for $c > 0$: $a \bullet c > b \bullet c$ and $a \div c > b \div c$.

If $a > b$, then for $c < 0$: $a \bullet c < b \bullet c$ and $a \div c < b \div c$.

Absolute value: $|a| = a$ if $a > 0$ and $-a$ if $a < 0$

If $|ax+b| = k$, then $ax+b = k$ or $ax+b = -k$. If $|ax+b| = |y|$, then $ax+b = y$ or $ax+b = -y$.

If $|ax+b| > k$, then $ax+b > k$ or $ax+b < -k$.

If $|ax+b| < k$, then $-k < ax+b < k$.

Graphs, Linear Equations, & Functions

Types of Linear Equations in one variable:

Conditional has one solution;
Identity has an infinite number of solutions;
Contradiction has no solution

Types of Linear Systems in two variables:

Graphs intersect at one point: Consistent and Independent - one solution;
Graphs are parallel lines: Inconsistent and Independent - no solutions;
Graphs lie on top of each other: Consistent and Dependent - infinite number of solutions

Mid-point of a line is $([x_1 + x_2] \div 2, [y_1 + y_2] \div 2)$

To find the **x-intercept**, let $y = 0$ and solve for x.

To find the **y-intercept**, let $x = 0$ and solve for y.

Slope of a line: $m = (y_2 - y_1) \div (x_2 - x_1)$

Positive sloped lines rise from left to right.
Negative sloped lines fall from left to right.
Parallel lines have equal slopes.
Perpendicular lines have slopes which are negative reciprocals (their product is -1).
Slope-Intercept form: $y = m \cdot x + b$ where m = slope and (0,b) is the y-intercept.
Point-Slope form: $(y - y_1) = m \cdot (x - x_1)$ where m = slope and (x_1, y_1) is a point on the line.
Standard form: $a \cdot x + b \cdot y = c$ where a, b, c are integers and $a > 0$
slope is $-a/b$; x-intercept is (c/a,0); y-intercept is (0,c/b)
Horizontal line: $y = b$ slope is zero and y-intercept is (0,b).
Vertical line: $x = a$ slope is undefined and x-intercept is (a,0).
Quadrants: I is upper right, II is upper left, III is lower left, IV is lower right
Domain is the set of first or independent values.
Range is the set of second or dependent values.
Distance Formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
Sum of functions: $(f+g)(x) = f(x) + g(x)$
Difference of functions: $(f-g)(x) = f(x) - g(x)$
Product of functions: $(f \cdot g)(x) = f(x) \cdot g(x)$
Quotient of functions: $(f \div g)(x) = f(x) \div g(x), g(x) \neq 0$

Exponents and Polynomials

Adding polynomials: Add like terms.
Subtracting polynomials: Change all the signs of the second polynomial and add to the first.
Multiplying polynomials: Multiply each term of the second polynomial by each term of the first polynomial and add the products.
The FOIL method:
1) Multiply the two **F**irst terms to get the first term of the answer.
2) Find the **O**uter and **I**nner product and combine them to get the middle term of the answer.
3) Multiply the two **L**ast terms to get the last term of the answer.

Product rule for exponents: $a^m \cdot a^n = a^{m+n}$
Quotient rule for exponents: $a^m \div a^n = a^{m-n}$
Power rule (a) for exponents: $(a^m)^n = a^{mn}$
Zero Exponent: $a^0 = 1$
Power rule (b) for exponents: $(ab)^m = a^m \cdot b^m$
Negative exponent: $a^{-n} = 1 \div a^n$
Power rule (c) for exponents: $(a/b)^m = a^m \div b^m$
Changing from negative to positive exponents: $a^{-m}/b^{-n} = b^n/a^m$ $(a/b)^{-m} = (b/a)^m$
Scientific Notation: $a \cdot 10^n$ where $1 \le a < 10$ and n is an integer e.g. $8000 = 8 \times 10^3$ & $0.0004 = 4 \times 10^{-4}$
Square of a binomial: $(a+b)^2 = a^2 + 2ab + b^2$ $(a-b)^2 = a^2 - 2ab + b^2$
Product of the sum and difference of two terms: $(a+b)(a-b) = a^2 - b^2$
Dividing a polynomial by a monomial: $(a+b) \div c = a \div c + b \div c$
Remainder Theorem: If a polynomial P(x) is divided by $(x - a)$, the remainder is equal to P(a).

Factoring

Finding the Greatest Common Factor:
1) Write each number in prime factored form.
2) List each prime number or each variable that is a factor of every term in the list.
3) The exponents on the prime factors are the *least* exponents from the prime factored forms.
4) Multiply these primes. If there are no primes left after 3), the greatest common factor is 1.

Factoring by grouping:
1) Collect the terms into two groups so that each group has a common factor.
2) Factor out the greatest common factor from each group.
3) Factor a common binomial factor from the result of 2).

4) If 2) does not result in a common binomial factor, try a different grouping.

Factoring x^2+bx+c:

1) Find two integers (m, n) whose product is *c* and whose sum is *b*.
2) Both integers must be positive if *b* and *c* are positive
3) Both integers must be negative if *c* is positive and *b* is negative.
4) One integer must be positive and one must be negative if *c* is negative.
5) Factors of the original expression are $(x \pm m)(x \pm n)$

Factoring ax^2+bx+c where $a \neq 1$:

1) Find two integers whose product is $a \bullet c$ and whose sum is *b*.
2) Both integers must be positive if *b* and *c* are positive
3) Both integers must be negative if *c* is positive and *b* is negative.
4) One integer must be positive and one must be negative if *c* is negative.
5) Rewrite the expression as $ax^2 \pm mx \pm nx + c$ and factor by grouping as above.

Factoring a difference of two squares: $a^2 - b^2 = (a+b)(a-b)$

[**Remember:** Sum of two squares is prime!]

Factoring perfect square trinomials: $a^2 + 2ab + b^2 = (a+b)^2$ $a^2 - 2ab + b^2 = (a-b)^2$

Factoring sum or difference of cubes: $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$ $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

General method for factoring a polynomial:

1) Is there a common factor?
2) If there are two terms, is it the difference of squares or the sum or difference of cubes?
3) If there are three terms, is it a perfect square?
4) If there are three terms and the coefficient of the squared term is 1, factor x^2+bx+c as above.
5) If there are three terms and the coefficient of the squared term is not 1, Separate the middle term into two terms with the same sum and then factor by grouping.
6) If there are four terms, try to factor by grouping.
7) Check if any of the factors can be factored further.

Zero-Factor Property: if $a \bullet b = 0$, then either $a = 0$ or $b = 0$ or both a and $b = 0$.

Solving a quadratic equation: Write it in standard form $(ax^2 + bx + c) = 0$ and factor completely. Set each factor to 0 and solve for x. Check each solution in the original equation.

Pythagorean formula: In a right triangle, the sum of the squares of the legs is equal to the square of the hypotenuse (= the side opposite the right angle; also = longest side). $a^2 + b^2 = c^2$

Rational Expressions

Least Common Denominator is the product of all of the different factors from each denominator, with each factor raised to the greatest power that occurs in any denominator.

Formulae:

Distance = rate • time

Interest = principal • rate • time

Rate of work = 1 ÷ t (= 1 job per unit of time)

Roots, Radicals and Root Functions

When n is even, principal root ($\sqrt[n]{a}$) is ≥ 0 and negative root ($-\sqrt[n]{a}$) is < 0.

When n is odd, there is only one root ($\sqrt[n]{a}$) and it can be + or -.

For odd n, $\sqrt[n]{a^n} = a$ For even n, $\sqrt[n]{a^n} = |a|$

$\sqrt[n]{a} = a^{1/n}, a \geq 0$

$\sqrt[n]{a^m} = (\sqrt[n]{a})^m = a^{m/n} = (a^{1/n})^m, a \geq 0$

$\sqrt[n]{a} \bullet \sqrt[n]{b} = \sqrt[n]{ab}$

$\sqrt[n]{a} \div \sqrt[n]{b} = \sqrt[n]{a/b}$, $b \neq 0$

Conjugate of the binomial $a + b$ is $a – b$

Conditions for a simplified radical:

1. The radicand has no factor raised to a power greater than or equal to the index.
2. The radicand has no fractions.
3. No denominator contains a radical.
4. Exponents in the radicand and the index of the radical have no common factors (except 1).

Perfect squares: $1^2=1$, $2^2=4$, $3^2=9$, $4^2=16$, $5^2=25$, $6^2=36$, $7^2=49$, $8^2=64$, $9^2=81$, $10^2=100$, $11^2=121$, $12^2=144$, $13^2=169$, $14^2=196$, $15^2=225$, $16^2=256$, $17^2=289$, $18^2=324$, $19^2=361$, $20^2=400$

Perfect cubes: $1^3=1$, $2^3=8$, $3^3=27$, $4^3=64$, $5^3=125$, $6^3=216$, $7^3=343$, $8^3=512$, $9^3=729$, $10^3=1000$, $11^3=1331$, $12^3=1728$

Perfect powers for index n: x^n, x^{2n}, x^{3n}, x^{4n}, x^{5n}, x^{6n}, …

Complex Numbers

$i = \sqrt{-1}$, $i^2 = -1$, $i^3 = -i$, $i^4 = 1$, $i^5=\sqrt{-1}$, $i^6 = -1$, $i^7 = -i$, $i^8 = 1$, ...

$\sqrt{-b} = i\sqrt{b}$

Complex number: $a + bi$ (real part plus imaginary part)

Quadratic Equations

Standard form of a Quadratic Equation: $ax^2 + bx + c = 0$, $a \neq 0$

Discriminant = $b^2 - 4ac$:

> 0 means 2 distinct, real solutions (rational if a perfect square, else irrational)

= 0 means 1 real, rational solution

< 0 means two complex solutions

If $x^2 = k$, then $x = \pm k$

Quadratic formula: $x = (-b \pm \sqrt{b^2 - 4ac}\ ) \div 2a$

Quadratic Function Form 1: $f(x) = ax^2 + bx + c$

Axis of symmetry: $x = -b/2a$

Vertex is at the point: $(-b\div 2a, [4ac-b^2] \div 4a)$

Quadratic Function Form 2: $f(x) = a(x - h)^2 + k$

Axis of Symmetry: $x = h$ **Y-intercept:** $(0,k)$

Vertex is at the point (h,k)

Parabola opens up if $a > 0$, down if $a < 0$

Exponentials and Logarithms

Exponential function: $y = a^x$ where $a > 0$, $a \neq 1$.

Solving exponential equations: if $a^x = a^y$, then $x = y$.

Logarithmic function: $y = \log_a x$ means $x = a^y$ where $a > 0$, $a \neq 1$, and $x > 0$

$\log_a a = 1$ $\log_a 1 = 0$

Graph of a logarithm passes through $(1\div a, -1)$, $(1, 0)$, and $(a, 1)$