Differentiation and Integration Rules

A derivative computes the instantaneous rate of change of a function at different values.

An indefinite integral computes the family of functions that are the antiderivative. A definite integral is used to compute the area under the curve

These are some of the most frequently encountered rules for differentiation and integration.

For the following, let u and v be functions of x, let n be an integer, and let a, c, and C be constants.

Fundamental Rules

$$\frac{d(c)}{dx} = 0$$

$$\int dx = x + C$$

$$\frac{d(c * u)}{dx} = c\,\frac{du}{dx}$$

$$\int c * u\,dx = c\int u\,dx$$

$$\frac{d(u+v)}{dx} = \frac{du}{dx} + \frac{dv}{dx}$$

$$\int u + v\,dx = \int u\,dx + \int v\,dx$$

$$\frac{d(x^n)}{dx} = n * x^{n-1}$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

$$\frac{d(uv)}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$$

$$\int u\,dv = uv - \int v\,du$$

$$\frac{d\left(\frac{u}{v}\right)}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

Trigonometric Functions

$$\frac{d(\sin x)}{dx} = \cos x$$

$$\int \cos x\,dx = \sin x + C$$

$$\frac{d(\cos x)}{dx} = -\sin x$$

$$\int \sin x\,dx = -\cos x + C$$

$$\frac{d(\tan x)}{dx} = sec^2 x$$

$$\int sec^2 x\,dx = \tan x + C$$

$$\frac{d(\cot x)}{dx} = -csc^2 x$$

$$\int csc^2 x\,dx = -\cot x + C$$

$$\frac{d(\sec x)}{dx} = \sec x \tan x$$

$$\int \sec x \tan x\,dx = \sec x + C$$

$$\frac{d(\csc x)}{dx} = -\csc x \cot x$$

$$\int \csc x \cot x\,dx = -\csc x + C$$

$$\int \tan x\,dx = ln\ |\sec x| + C$$

$$\int \cot x\,dx = ln|\sin x| + C$$

Exponential and Logarithmic Functions

$$\frac{d(e^u)}{dx} = e^u \frac{du}{dx} \qquad \int e^x dx = e^x + C$$

$$\frac{d(\ln u)}{dx} = \frac{1}{u}\frac{du}{dx} \qquad \int \frac{1}{x} dx = ln|x| + C$$

$$\frac{d(a^u)}{dx} = (a^u)(\ln a)\frac{du}{dx} \qquad \int a^x dx = \frac{a^x}{\ln a} + C$$

$$\frac{d(u^v)}{dx} = u^v(v' \ln u + \frac{v}{u}u')$$

Inverse Trigonometric Functions

$$\frac{d(sin^{-1}x)}{dx} = \frac{1}{\sqrt{1-x^2}} \qquad \int \frac{1}{\sqrt{a^2-x^2}} dx = sin^{-1}\frac{x}{a} + C$$

$$\frac{d(cos^{-1}x)}{dx} = \frac{-1}{\sqrt{1-x^2}} \qquad \int \frac{-1}{\sqrt{a^2-x^2}} dx = -sin^{-1}\frac{x}{a} + C$$

$$\frac{d(tan^{-1}x)}{dx} = \frac{1}{1+x^2} \qquad \int \frac{1}{a^2+x^2} dx = \frac{1}{a}tan^{-1}\frac{x}{a} + C$$

$$\frac{d(cot^{-1}x)}{dx} = \frac{-1}{1+x^2} \qquad \int \frac{-1}{a^2+x^2} dx = -\frac{1}{a}tan^{-1}\frac{x}{a} + C$$

$$\frac{d(sec^{-1}x)}{dx} = \frac{1}{|x|\sqrt{x^2-1}} \qquad \int \frac{1}{|x|\sqrt{x^2-a^2}} dx = \frac{1}{a}sec^{-1}\left|\frac{x}{a}\right| + C$$

$$\frac{d(csc^{-1}x)}{dx} = \frac{-1}{|x|\sqrt{x^2-1}} \qquad \int \frac{-1}{|x|\sqrt{x^2-a^2}} dx = -\frac{1}{a}sec^{-1}\left|\frac{x}{a}\right| + C$$

Hyperbolic Trigonometric Functions

$$\frac{d(\sinh x)}{dx} = \cosh x \qquad \int \cosh x\, dx = \sinh x + C$$

$$\frac{d(\cosh x)}{dx} = \sinh x \qquad \int \sinh x\, dx = \cosh x + C$$

$$\frac{d(\tanh x)}{dx} = sech^2 x \qquad \int \tanh x\, dx = -ln|\operatorname{sech} x| + C$$

Inverse Hyperbolic Trigonometric Functions

$$\frac{d(sinh^{-1}x)}{dx} = \frac{1}{\sqrt{1+x^2}} \qquad \int \frac{1}{\sqrt{1+x^2}} dx = sinh^{-1}x + C$$

$$\frac{d(cosh^{-1}x)}{dx} = \frac{1}{\sqrt{x^2-1}} \qquad \int \frac{1}{\sqrt{x^2-1}} dx = cosh^{-1}x + C$$

$$\frac{d(tanh^{-1}x)}{dx} = \frac{1}{1-x^2} \qquad \int \frac{1}{1-x^2} dx = tanh^{-1}x + C$$