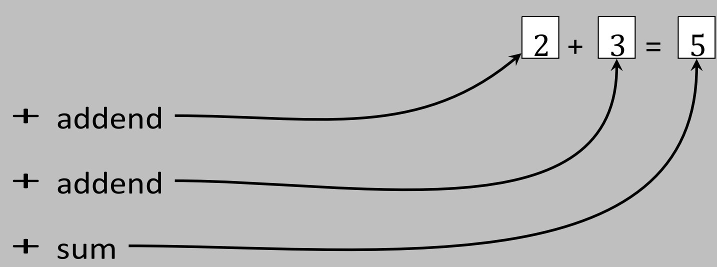
Whole Numbers

Whole numbers are the building blocks of all mathematics. They are the familiar sequence that starts as 0,1,2.

Adding Whole Numbers

When we add two numbers together, the numbers being added are called "**addends**" and what they add up to is called the "**sum**".



Here are some properties of addition:

- The **addition property of 0** says that the sum of 0 and any number is that number. For example $0 + 2 = 2$ and $2 + 0 = 0$.
- The **commutative property of addition** says that changing the order of two numbers being added does not change their sum.
  For example $3 + 4 = 7$ and $4 + 3 = 7$.
- The **associative property of addition** says that if we add three numbers, putting parentheses around the first two or the second two doesn't change the sum.
  For example $(2 + 3) + 4 = 5 + 4 = 9$ and $2 + (3 + 4) = 2 + 7 = 9$.

Subtracting Whole Numbers

When we subtract two numbers, the result is called the "**difference**".

6 − 2 = 4

+ difference

Here are some properties of subtraction:

+ The difference of any number and itself is 0. For example 4 − 4 = 0. Also 0 − 0 = 0.
+ The difference of any number and 0 is itself. For example 4 − 0 = 4.

CAUTION: order matters in subtraction, so 4 − 2 is not the same as 2 − 4.

Multiplying Whole Numbers

Multiplication is repeated addition.

$$3+3=2\cdot 3=6$$
$$3+3+3=3\cdot 3=9$$
$$3+3+3+3=4\cdot 3=12$$

In algebra, × is not used because it can be confused for the variable x. Another way to write the product of two numbers is by putting one next to the other with the second one in parentheses.

$$2\cdot 3=2(3)=6$$

The result of multiplying two numbers is called the "**product**". The numbers being multiplied are "**factors**" of the product.

$$2\cdot 3=6$$

+ factor
+ factor
+ product

Here are some properties of multiplication:

+ The **multiplication property of 0** says that the product of 0 and any number is 0.
  For example $3 \cdot 0 = 0$ and $0 \cdot 3 = 0$.
  Also $0 \cdot 0 = 0$.

+ The **multiplication property of 1** says that the product of 1 and any number is that same number. For example:

$$4 \cdot 1 = 4 \text{ and } 1 \cdot 4 = 4$$

  We still have

$$0 \cdot 1 = 0 \text{ and } 1 \cdot 0 = 0$$

  Also

$$1 \cdot 1 = 1$$

+ The **commutative property of multiplication** says that changing the order of two numbers being multiplied doesn't change their product. For example:

$$4 \cdot 3 = 3 + 3 + 3 + 3 = 12 \quad \text{and} \quad 3 \cdot 4 = 4 + 4 + 4 = 12$$

+ The **associative property of multiplication** says that changing the grouping of factors does not change their product. For example:

$$4 \cdot (3 \cdot 5) = 4 \cdot 15 = 60$$

$$(4 \cdot 3) \cdot 5 = 12 \cdot 5 = 60$$

+ The **distributive property** says that multiplication distributes over addition. For example:

$$3 \cdot (4 + 5) = 3 \cdot 4 + 3 \cdot 5$$

  Also:

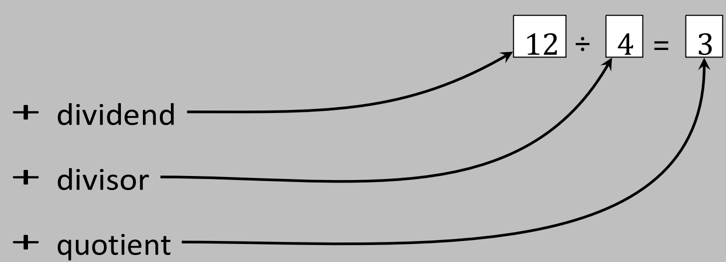
$$4 + 5) \cdot 3 = 4 \cdot 3 + 5 \cdot 3$$

Dividing Whole Numbers

Division means separating a quantity into equal parts.

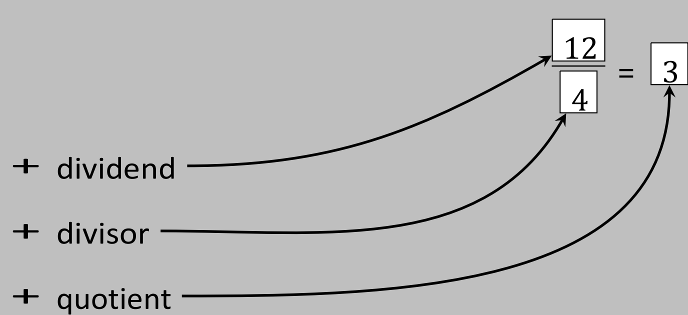
Be careful what part goes where. The number that is being divided is called the **dividend** and the number that it is being divided into is called the **divisor**. The number of times the divisor goes into the dividend is called the **quotient**.

There are three different ways to write a division problem. It is important to understand each way and what part goes where. Let's say we want to divide 12 by 4. Then 12 is the divisor and 4 is the dividend. We could use the symbol $\div$ for division:



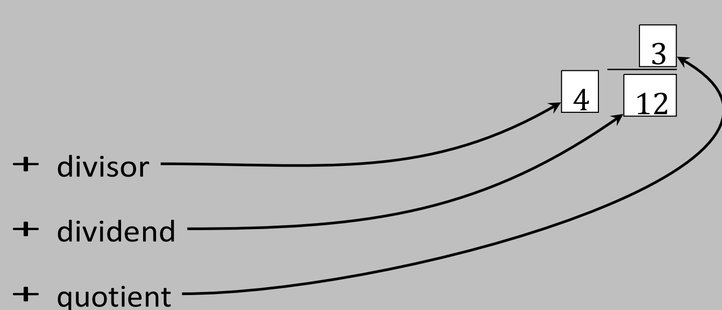
In words we would say "12 divided by 4 is 3."

We could also use a bar to represent division. Note that **the dividend goes on top and the divisor goes on the bottom**. We would write:



We would read this "12 over 4 is 3."

We use the following for long division:



This is read "4 into 12 is 3." Note: the dividend goes on the **outside** when using this notation and the divisor goes on the **inside**.

Some properties:

+ The quotient of 0 and any number (except 0) is 0.

$$\frac{0}{5} = 0 \qquad 6\overline{)0}^{\,0} \qquad 0 \div 7 = 0$$

+ The quotient of any number and 0 is not a number. For example:

$$\frac{3}{0} \qquad 0\overline{)\ 6} \qquad 4 \div 0$$

are **undefined**.

Exponents

We use **exponents** for repeated multiplication.

$$\underbrace{3 \cdot 3 \cdot 3 \cdot 3 \cdot 3}_{\text{3 is a factor 5 times}} = 3^5$$

+ base (3)
+ exponent (5)

Reading:

+ $4 = 4^1$ is "four to the first power."
  Note: we don't need to write the exponent when it is 1.
+ $4 \cdot 4 = 4^2$ is read "four to the second power" or " four squared."
+ $4 \cdot 4 \cdot 4 = 4^3$ is read as "four to the third power" or "four cubed."
+ $4 \cdot 4 \cdot 4 \cdot 4 = 4^4$ is read as "four to the fourth power." There are no special ways to read powers higher than 3.

Order of Operations

When using more than one operation in the same expression, we need to be careful to do the operations in the correct order.

1. Perform all operations within grouping symbols such as parentheses () or brackets [].
2. Evaluate any expressions with exponents.
3. Multiply or divide in order from left to right.
4. Add or subtract in order from left to right.

Example: We are given

$$6 \cdot 2 \div 4 + 3^2 - 4$$

First, we evaluate the exponent $3^2 = 9$ which gives:

$$6 \cdot 2 \div 4 + 9 - 4$$

Next, we evaluate the multiplications and divisions in order $6 \cdot 2 \div 4 = 12 \div 4 = 3$ which gives:

$$3 + 9 - 4$$

Finally, do all additions and subtractions from left to right. If we put $*$ under an operation, that's the one we're doing:

$$\underbrace{3+9}_{*}-4$$

$$\underbrace{12-4}_{*}$$

$$8$$

So:

$$6 \cdot 2 \div 4 + 3^2 - 4 = 8$$

It's extremely common to see this written as:

$$\frac{6 \cdot 2}{4} + 3^2 - 4$$

When something is in parentheses, it's important to follow order of operations for everything that's inside before moving outside.

Example: $\left(2\cdot 3-\frac{6}{3}\right)\cdot 2-5$ We do what's inside parentheses first. There is one multiplication and one division.

$$\left(\underbrace{2\cdot 3}_{*}-\frac{6}{3}\right)\cdot 2-5$$

$$\left(6-\underbrace{\frac{6}{3}}_{*}\right)\cdot 2-5$$

$$\left(\underbrace{6-2}_{*}\right)\cdot 2-5$$

$$4\cdot 2-5$$

Now that we are done with parentheses, we finish the problem still using order of operations.

$$\underbrace{4\cdot 2}_{*}-5$$

$$\underbrace{8-5}_{*}$$

$$3$$

Sometimes there are things you should consider to "be in parentheses" even if there are no actual parentheses.

+ Anything above or below a division line is automatically considered "in parentheses."

Example: $\frac{8-4}{5-3}$

$$\frac{\overbrace{8-4}^{*}}{5-3}$$

$$\frac{4}{\underbrace{5-3}_{*}}$$

$$\underbrace{\frac{4}{2}}_{*}$$

$$2$$

+ Anything in the exponent is considered “in parentheses.”

Example: 2^{4-1}

$$2^{\overbrace{4-1}^{*}}$$

$$\underbrace{2^{3}}_{*}$$

$$8 \quad \{$$

Now Give It a Try!

1. $2 \cdot 3^2 - 5$

2. $(3 - 2) \cdot 3 + 5$

3. $$20 - (2 + 3) \cdot 2 + \frac{6}{2} - 3$$

4. $[2 \cdot (3 + 2) + 1] - 11$

5. $2^{2 \cdot 4-6}$

Answerkey:

1. 13
2. 8
3. 8
4. 0
5. 4