Truth Tables

Truth tables are used to determine the validity or truth of a compound statement*.

- A compound statement is composed of one or more simple statements. Simple statements are typically represented by symbols (often letters).

- Each symbol represents a statement such as "John scored a goal" or "It is raining."

- Constructing a truth table for a compound statement depends upon the simple statements composing the compound statement

*The term statement may also be referred to as a premise or expression depending on the context.

To construct a truth table for a compound statement that consists of <u>two simple statements</u>, begin by listing the four true-false cases shown below:

p	q
T	T
T	F
F	T
F	F

If we use the example statements above, we can see what this would translate to.

John scored a goal.	It is raining.
T	T
T	F
F	T
F	F

In the first row, John scored a goal, and it is raining are true. Both events occurred.

To construct a truth table for a compound statement that consists of <u>three simple statements</u>, begin by listing the eight true-false cases shown below:

p	q	r
T	T	T
T	T	F
T	F	T
T	F	F
F	T	T
F	T	F
F	F	T
F	F	F

The number of times T appears consecutively in each column is determined by the number of statements.

In this example, the formula for the number of times T is listed consecutively under statement p is 2^2, the formula for the number of times T is listed consecutively under statement q is 2^1, and the formula for the number of times T is listed consecutively under statement r is 2^0.

One good way to remember the sequence is to remember that the number of times T is listed consecutively in the first statement is half the number of true-false cases. For the next statement, it would be half of the first, and so on.

Note that the number of true-false cases has doubled with the addition of one statement. The total number of cases will be determined by using 2 to the power of the number of statements. This means that if there are four statements, we would determine the number of cases by calculating 2^4.

Connectives

The validity or truth of the compound statement is dependent upon the simple statements and the connective used. Connectives are symbols that indicate the relationship between simple statements.

The five most common connectives are listed below.

If multiple connectives are used, the truth of the connectives must be completed in a particular order (see below for details).

In the following examples, **p** represents the statement "John scored a goal" and **q** represents the statement "John won the game."

Negation - " Not " - Complete *First*

p	~p
T	F
F	T

The statement is true when the input statement is false.
The statement is false when the input statement is true.

~p represents the statement "John did **not** score a goal."

Conjunction – "and" - Complete *Second* along with Disjunction

p	q	p Λ q
T	T	T
T	F	F
F	T	F
F	F	F

The statement is **true only when <u>both</u> input statements are true**.
Otherwise, the statement is false.

p ^ q represents the statement "John scored a goal **and** won the game."

Disjunction – "or" - Complete *Second* along with Conjuction

p	q	p V q
T	T	T
T	F	T
F	T	T
F	F	F

The statement is **false only when <u>both</u> input statements are false**.
Otherwise, the statement is true.

p V q represents the statement "John scored a goal **or** won the game."

Conditional – "if ….. then" - Complete *Third*

p	q	p → q
T	T	T
T	F	F
F	T	T
F	F	T

The statement is **false only when the first input statement is true, and the second input statement is false**.
Otherwise, the statement is true.

p→q represents the statement "**If** John scored a goal, **then** won the game."

Biconditional – "if and only if" - Complete *Last*

P	q	p ↔ q
T	T	T
T	F	F
F	T	F
F	F	T

The statement is **true when both input statements are both true, or both false.**
Otherwise, the statement is false.

p ↔ q represents the statement "John scored a goal **if and only if** he won the game."