# Summary of Convergence and Divergence Tests for Infinite Series

| Test | Series | Convergence or Divergence | Comments | When to Use Strategies |
|---|---|---|---|---|
| ***n*th term test for divergence** | $\sum_{n=1}^{\infty} a_n$ | Diverges if $\lim_{n\to\infty} a_n \neq 0$ | Inconclusive if $\lim_{n\to\infty} a_n = 0$ | If, at glance, $\lim_{n\to\infty} a_n \neq 0$, then use *n*th term test |
| **Geometric Series** | $\sum_{n=1}^{\infty} ar^{n-1}$ | Converges if $\lvert r\rvert < 1$<br><br>Diverges if $\lvert r\rvert \geq 1$ | If the series converges, $\sum_{n=1}^{\infty} ar^{n-1} = \frac{a}{1-r}$ | If series has form $\sum_{n=1}^{\infty} ar^{n-1}$ or $\sum_{n=1}^{\infty} ar^{n}$ |
| **p-series** | $\sum_{n=1}^{\infty} \frac{1}{n^p}$ | Converges if $p > 1$<br><br>Diverges if $p \leq 1$ | | If series has form $\sum_{n=1}^{\infty} \frac{1}{n^p}$ |
| **Integral Test** | $\sum_{n=1}^{\infty} a_n$<br><br>$a_n = f(n)$ | Converges if $\int_1^{\infty} f(n)\,dn$ converges<br><br>Diverges if $\int_1^{\infty} f(n)\,dn$ diverges | The function *f(n)* must be a continues, positive, and decreasing. | If $a_n = f(n)$, and $\int_1^{\infty} f(n)\,dn$ is easily evaluated. Hypotheses must be satisfied |
| **Individual Comparison Test** | $\sum_{n=1}^{\infty} a_n$ given<br><br>$\sum_{n=1}^{\infty} b_n$ comparison series | If $\sum_{n=1}^{\infty} b_n$ converges and $a_n \leq b_n$ for every n, then $\sum_{n=1}^{\infty} a_n$ also converges<br><br>If $\sum_{n=1}^{\infty} b_n$ diverges and $b_n \leq a_n$ for every n, then $\sum_{n=1}^{\infty} a_n$ also diverges | The comparison series ($\sum_{n=1}^{\infty} b_n$) must be supplied by you.<br><br>To prove convergence, the comparison series must converge and be a larger series.<br><br>To prove divergence, the comparison series must diverge and be a smaller series | If the series has a form similar to that of a p-series or geometric series. In particular, if $a_n$ is a rational function or is algebraic (features roots of polynomials), then the series should be compared to a p-series. |
| **Limit Comparison Test** | $\sum_{n=1}^{\infty} a_n$ given<br><br>$\sum_{n=1}^{\infty} b_n$ comparison series | If $\lim_{n\to\infty} \frac{a_n}{b_n} = L$ for some positive real number *L*, then both series converge or both diverge. | The comparison series ($\sum_{n=1}^{\infty} b_n$) must be supplied by you.<br><br>To prove convergence, the comparison series must converge<br><br>To prove divergence, the comparison series must diverge | If determining a bounded sequence proves to be difficult, then Limit Comparison Test should be used over the Individual Comparison Test |

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| **Ratio Test** | $\sum_{n=1}^{\infty} a_n$ | $\lim_{n\to\infty}\left\lvert\frac{a_{n+1}}{a_n}\right\rvert = L$ If $L < 1$ then the series converges absolutely If $L > 1$ $(or\ \infty)$ then the series diverges | Inconclusive if $L = 1$ | Series that involve factorials or other products, such as a constant raised to the $n$th power |
| **Root Test** | $\sum_{n=1}^{\infty} a_n$ | $\lim_{n\to\infty}\sqrt[n]{a_n} = L$ If $L < 1$ then the series converges absolutely If $L > 1$ $(or\ \infty)$ then the series diverges | Inconclusive if $L = 1$. Useful when the nth term involves nth powers. | If $a_n$ is of the form $(b_n)^n$ |
| **Alternating Series test** | $\sum_{n=1}^{\infty}(-1)^n a_n$ | Converges if: $a_n \geq 0$ $a_{n+1} \leq a_n$ $\lim_{n\to\infty} a_n = 0$ | Applies only to alternating series. | If of the form $\sum_{n=1}^{\infty}(-1)^n a_n$ or $\sum_{n=1}^{\infty}(-1)^{n-1} a_n$ |
| **Absolute Convergence** | $\sum_{n=1}^{\infty} a_n$ | If $\sum_{n=1}^{\infty}\lvert a_n\rvert$ converges then $\sum_{n=1}^{\infty} a_n$ converges absolutely. | If a series converges but does not converge absolutely, it conditionally converges. | Used if there are negative terms present in the series |
| **Telescoping Series** | $\sum_{k=1}^{\infty}\frac{1}{k(k+1)}$ | $S_n = \sum_{k=1}^{n}\frac{1}{k(k+1)}$ Converges if $\lim_{n\to\infty} S_n = \mathbb{R}$ | Can be rewritten using partial fraction decomposition. When finding the partial sum, many of the terms will cancel out. | No set form. Be careful to make sure all terms will cancel before determining convergence or divergence. |