Ratios and Proportions

A ratio is a relationship between two numbers. It indicates how many of the first number is included in the second number. Ratios can be written in three different ways: words, fractional notation, and colon notation.

Example: A recipe calls for 1 cup of milk to 3 cups of flour.

> Words: 1 to 3
>
> Fractional Notation: $\frac{1}{3}$
>
> Colon Notation: 1 : 3

Writing a Ratio as a Fraction

> The order of the quantities in a ratio is important. In order to write a ratio as a fraction, use the following steps.
>
> **Step 1:** Write the first number in the ratio in the numerator **Step 2:** Write the second number in the denominator

Example:
> Write the ratio 2 to 3 as a fraction.
> $$\frac{2}{3}$$

Hint:

> + The order of the numbers is very important. The ratio 2 to 3 is $\frac{2}{3}$. The fraction $\frac{3}{2}$ is incorrect.

Simplifying Ratios

> Ratios can be simplified by writing them in lowest terms. In order to do so, use the following steps.
>
> **Step 1:** Write the ratio as a fraction
> **Step 2:** Reduce the fraction to lowest terms **Step 3:** Rewrite the fraction as a ratio

Example: Write the ratio 4 to 6 in simplest form.

+ First, we write the ratio as a fraction:

$$\frac{4}{6}$$

+ Second, we reduce the fraction to lowest terms:

$$\frac{4}{6}=\frac{2}{3}$$

+ Third, we rewrite the new fraction as a ratio:

2 to 3

Rates

A special type of ratio is a rate.

Rates are used to compare different kinds of quantities. For example, you can purchase 3 boxes of cereal for 5 dollars. This can be written as follows.

$$\frac{3 \text{ boxes}}{5 \text{ dollars}}$$

Hint:

+ When comparing quantities with different units, write out the units as part of the ratio. They do no cancel out.

Same Units: $\frac{3 \text{ dollars}}{6 \text{ dollars}}=\frac{1}{2}$

Different Units: $\frac{3 \text{ dollars}}{6 \text{ box}}=\frac{1 \text{ dollars}}{2 \text{ box}}$

Unit Rate

A unit rate is a rate with a denominator of 1. A common example of a unit rate is driving speed. For example, 20 mph, read as "20 miles per hour" can be written as follows.

$$\frac{20 \text{ miles}}{1 \text{ hour}}$$

In order to write a rate as a unit rate, use the following steps.

Step 1: Write the rate as a fraction

Step 2: Divide the numerator by the denominator

Example:

A trucker drove 100 miles in 2 hours. Find the unit rate.

+ First, we write the rate as a fraction:

100 miles

$$\frac{\quad\quad}{2 \text{ hours}}$$

+ Second, we reduce the fraction to lowest terms:

$$\frac{100 \text{ miles}}{2 \text{ hours}} = \frac{50 \text{ miles}}{1 \text{ hours}}$$

The trucker is driving at a rate of 50mph

Proportions

A proportion is an equation stating that two ratios or rates are equal. It is written in the following form.

$$\frac{a}{b} = \frac{c}{d}$$

If this equation is true, than the two ratios are equivalent.

This proportion can also be read as "a is to b as c is to d." The ratios are separated by the word "as."

Cross Products

A cross product, also known as cross multiplying, is a technique that can be used to determine whether a proportion is true or to solve an equation. A cross product can be performed using the following steps.

Step 1: Write out the proportion

$$\frac{a}{b} = \frac{c}{d}$$

Step 2: Find the product of "a" and "d" and set that equal to the product of "b" and "c"

$$\frac{a}{b} = \frac{c}{d}$$

$$a * d = b * c$$

Hints:

+ You can think of a cross product as multiplying on a diagonal across the equals sign.

+ If the cross products are equal, then the proportion is true

Example:

Is the following proportion true?

$$\frac{3}{5} = \frac{9}{15}$$

+ First, we perform the cross product:

$$3 * 15 = 9 * 5$$

+ Second, we simplify the equation:

$$45 = 45$$

The original proportion is true.

Problem Solving using Proportions

Writing proportions can be used to solve various word problems. If given a ratio or rate of two quantities, a proportion can be used to determine an unknown quantity. In order to do so, use the following steps.

Step 1: Translate the word problem into a proportion, using x as the unknown quantity.

Step 2: Find the cross product

Step 3: Solve the equation

Step 4: Interpret the answer

Hint:

+ Remember that, when writing the proportion, both numbers in the numerator must have the same units. Both numbers in the denominator must have the same units as well.

Example:

It takes 5 cups of flour to make 3 batches of cookies. If you want to make 4 batches of cookies, how many cups of flour will you need?

+ First, we write out the word problem as a proportion:

$$\frac{5 \text{ cups of flour}}{3 \text{ batches of cookies}} = \frac{x \text{ cups of flour}}{4 \text{ batches of cookies}}$$

+ Second, we cross multiply:

$$5 \text{ cups} * 4 \text{ batches} = x \text{ cups} * 3 \text{ batches}$$

+ Third, we solve for x:

$$x \text{ cups} = 6\frac{2}{3} \text{ cups}$$

+ Fourth, we interpret the results:

We need $6\frac{2}{3}$ cups of flour.

Now Give It a Try!

1. Write the ratio 5 to 6 as a fraction

2. Write the fraction $\dfrac{7}{11}$ as a ratio in colon notation

3. Write the ratio 4 to 3 as a fraction

4. Write the ratio 4 to 8 in simplest terms

   Rewrite the following rates as a unit rate.

5. 100 passengers to 5 trains

6. 3 boys to 2 girls

7. 1 tank of gas to 40 miles

   Are the following proportions true?

8. $\dfrac{5}{20} = \dfrac{3}{12}$

9. $\dfrac{9}{3} = \dfrac{1}{3}$

10. It takes 3 hours to drive 180 miles. How long will it take to drive 330 miles?